What do frames and the medial surface tell us about decomposition for hex meshing?

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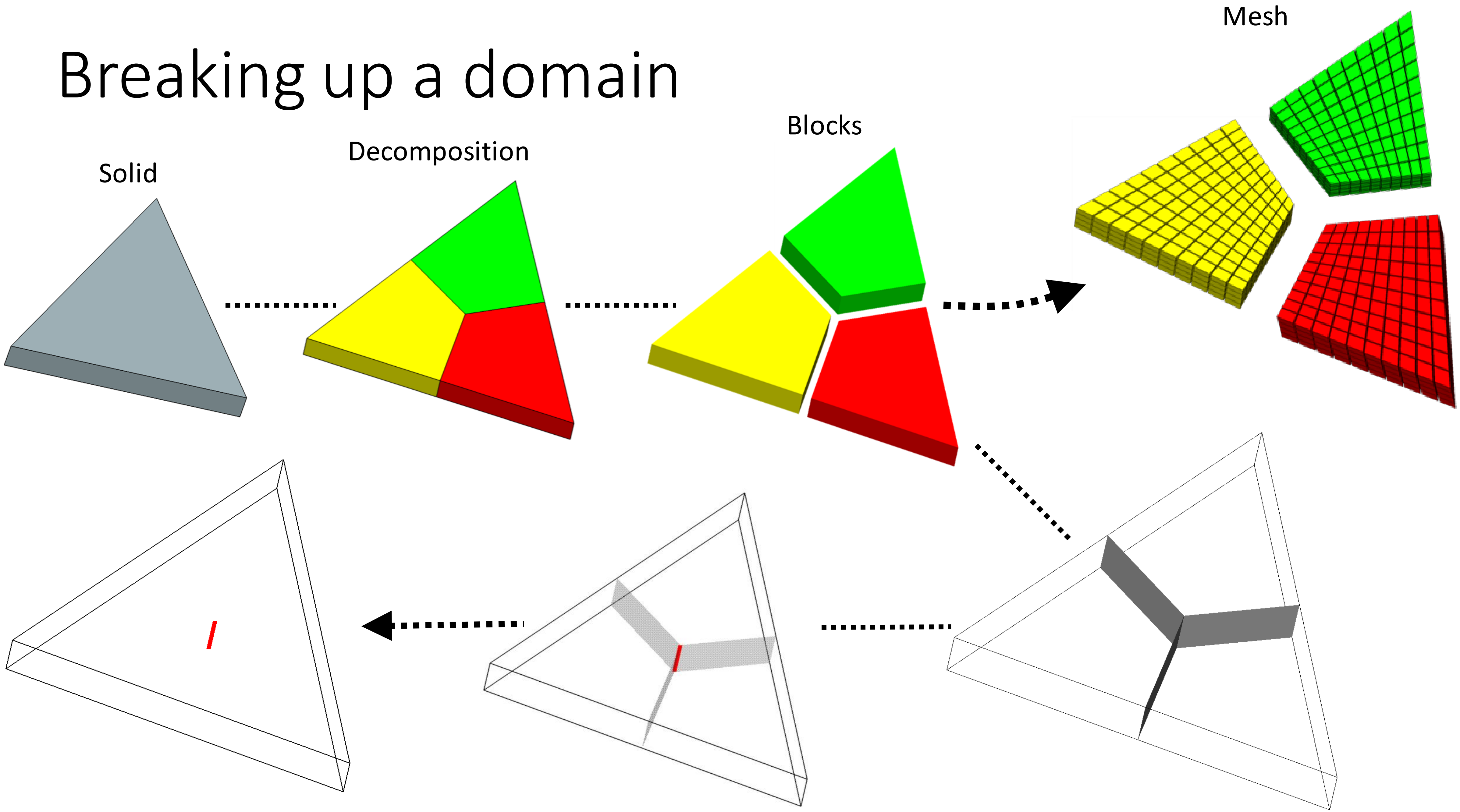
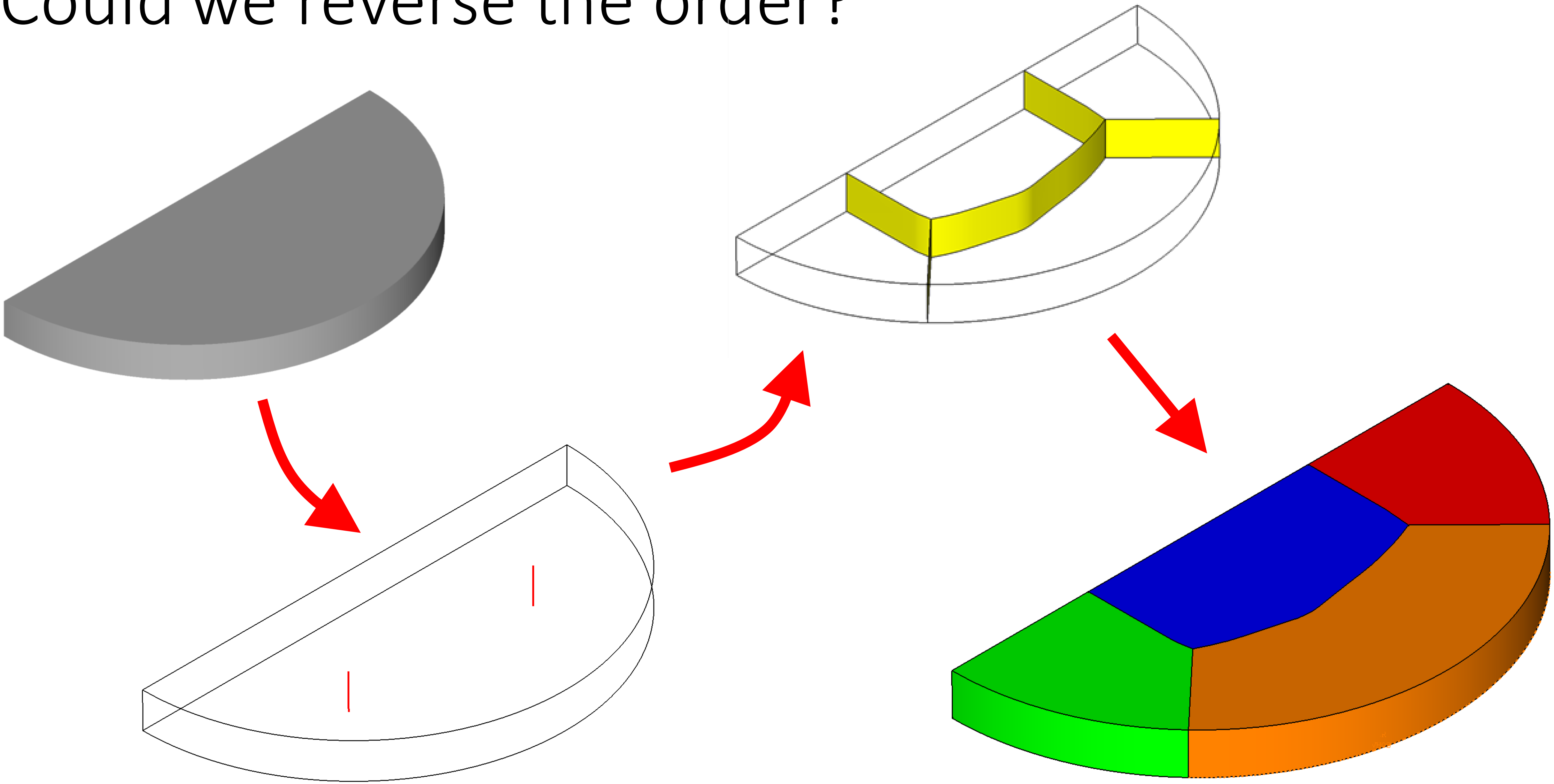
Breaking up a domain

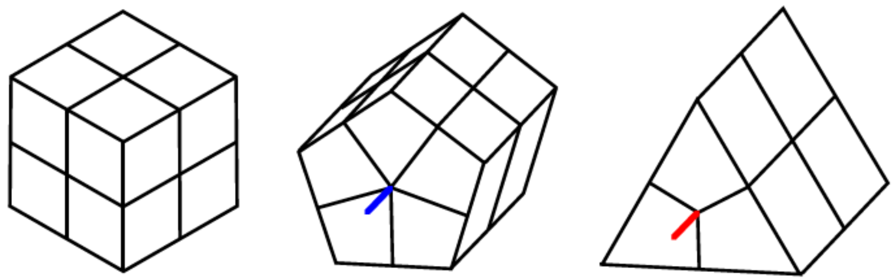


Could we reverse the order?



Singularity lines (hexahedral meshes)

- On a mesh: a collection of connected mesh edges where more or less than four mesh elements join.
- On a decomposition: Curves where more or less than four partition surfaces join.
- Types
  1. Negative (3 elements / partition surfaces)
  2. Positive (5 elements / partition surfaces)



Connectivity on the interior

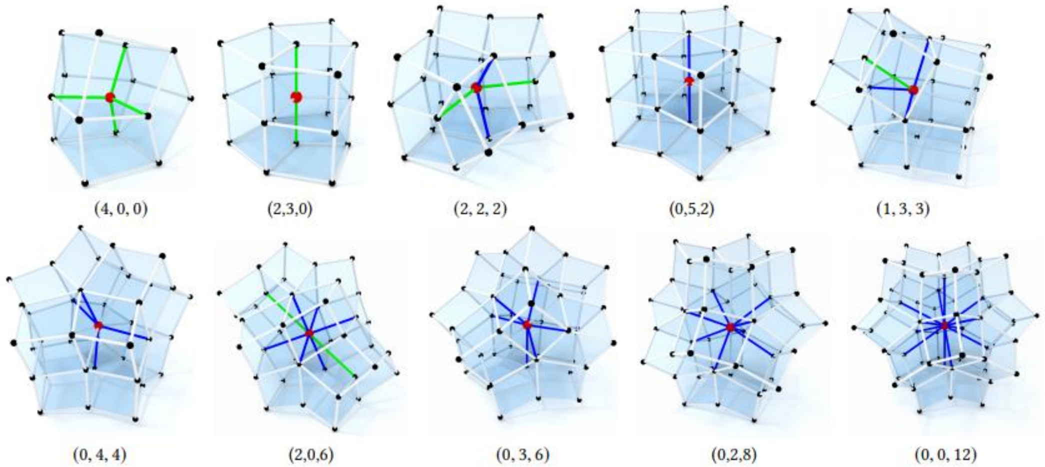
- Fundamental properties studied by Price et al.*
- Singularities join on internal vertices only in a certain number of configurations.
- Hex elements on these vertices join to convex polygons that satisfy

$$3F_3 + 2F_4 + F_5 = 12 \,, \quad \begin{array}{l} F_3 \; number\ of\ 3-sided\ faces \\ F_4 \; number\ of\ 4-sided\ faces \\ F_5 \; number\ of\ 5-sided\ faces \end{array}$$

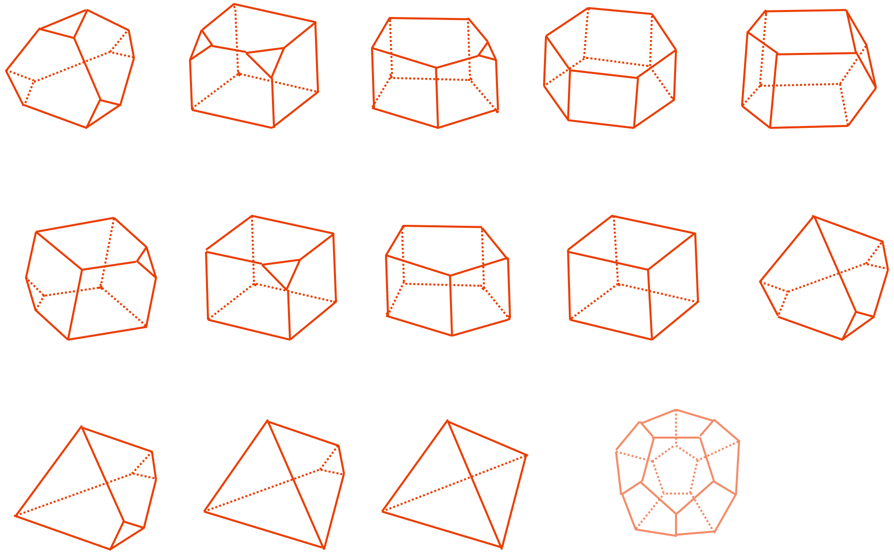
M. A. Price, C. G. Armstrong, M. A. Sabin. 1995. Hexahedral Mesh Generation by Medial Surface Subdivision: Part I. Solids with Convex Edges. International Journal for Numerical Methods in Engineering, Vol. 38, 3335-3359

Singularity lines

- In 3D singularities either
    i. End up on 2D singularities on the boundary
    ii. Connect to other singularities
    iii. Form loops with themselves
- Positive and negative singularities can connect only in certain configurations



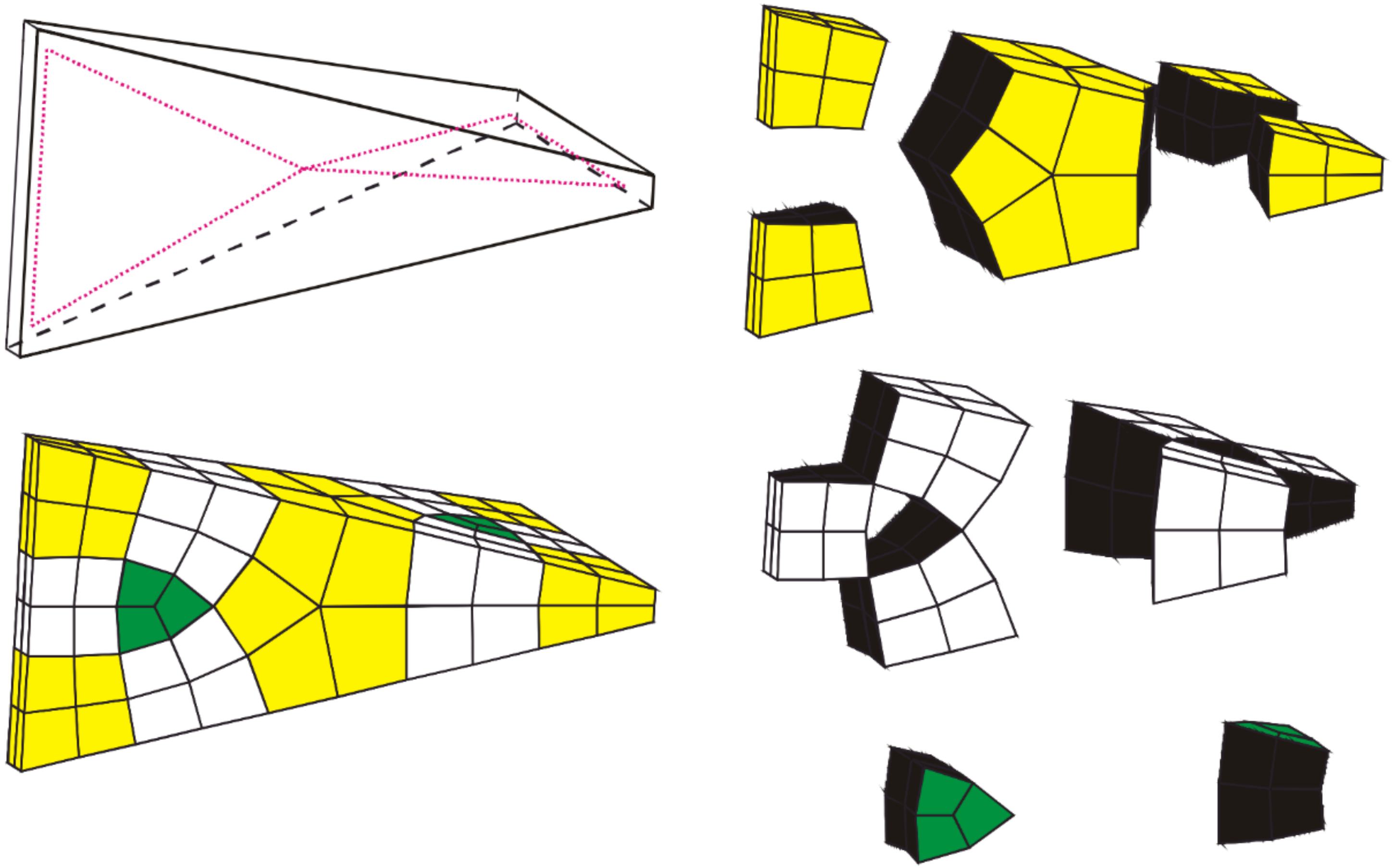
Heng Liu, Paul Zhang, Edward Chien, Justin Solomon, David Bommes. 2018. Singularity-Constrained Octahedral Fields for Hexahedral Meshing. ACM Trans. Graph. 37, 4, Article 93 (August 2018), 17 pages.



M. A. Price, C. G. Armstrong, M. A. Sabin. 1995. Hexahedral Mesh Generation by Medial Surface Subdivision: Part I. Solids with Convex Edges. International Journal for Numerical Methods in Engineering, Vol. 38, 3335-3359

Example

- Primitives used to decompose the domain.



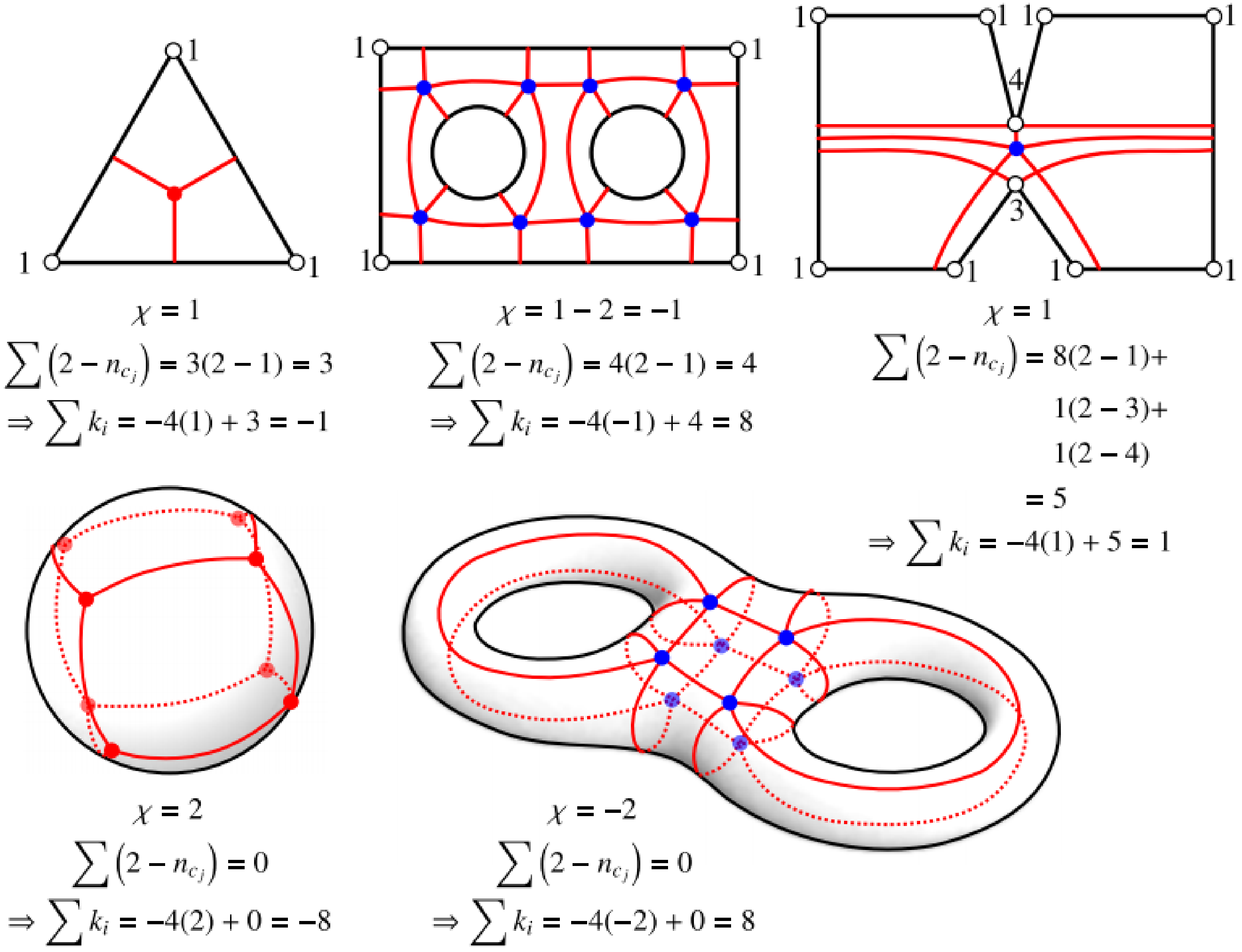
Boundary conformity

- Boundary faces impose constrains on the hex mesh topology.
- For each face R the net sum of singularity indices is

$$P_k = -4x(R) + \sum_{j=1}^{\#V} \left(2 - n_{c_j}\right)$$

where $x(R)$ is the Euler characteristic and n_{c_j} is the classification of vertex j.

Examples



Fogg HJ, Sun L, Makem J, Armstrong C, "Singularities in structured meshes and cross-fields," ***Comput. Des.*****, vol. 105, pp. 11–25, 2018**

Current state of the art

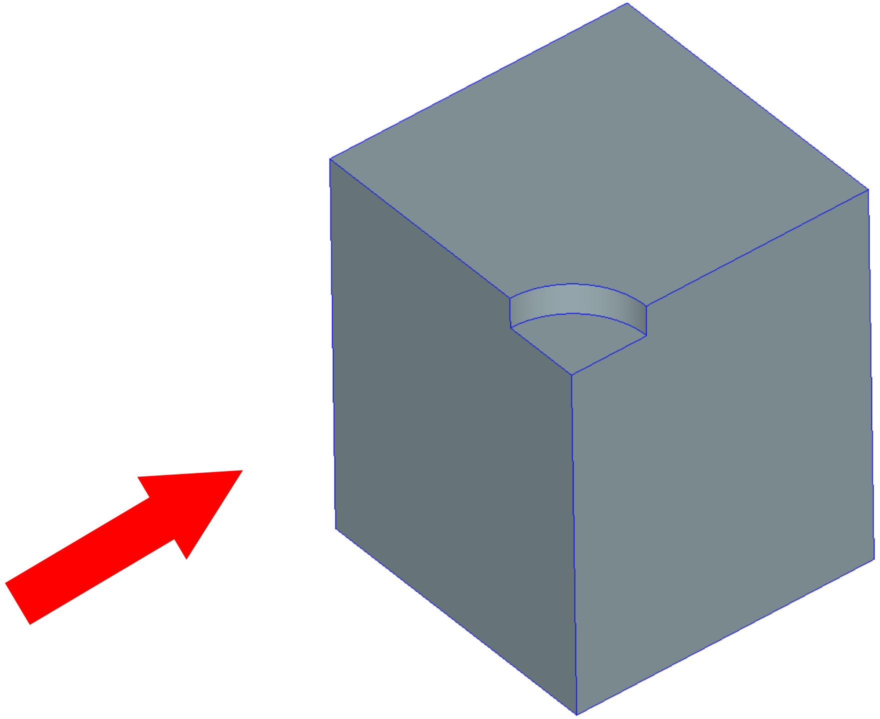
- Trying to address both boundary and internal constraints.
  - Create a 2D cross-field on the boundary.
    - Possibly extend to a 3D frame-field on the interior.
      - Identify and correct singularity network.
    - Or create loops on the boundary
  - Parameterization/Surface generation

Pros:

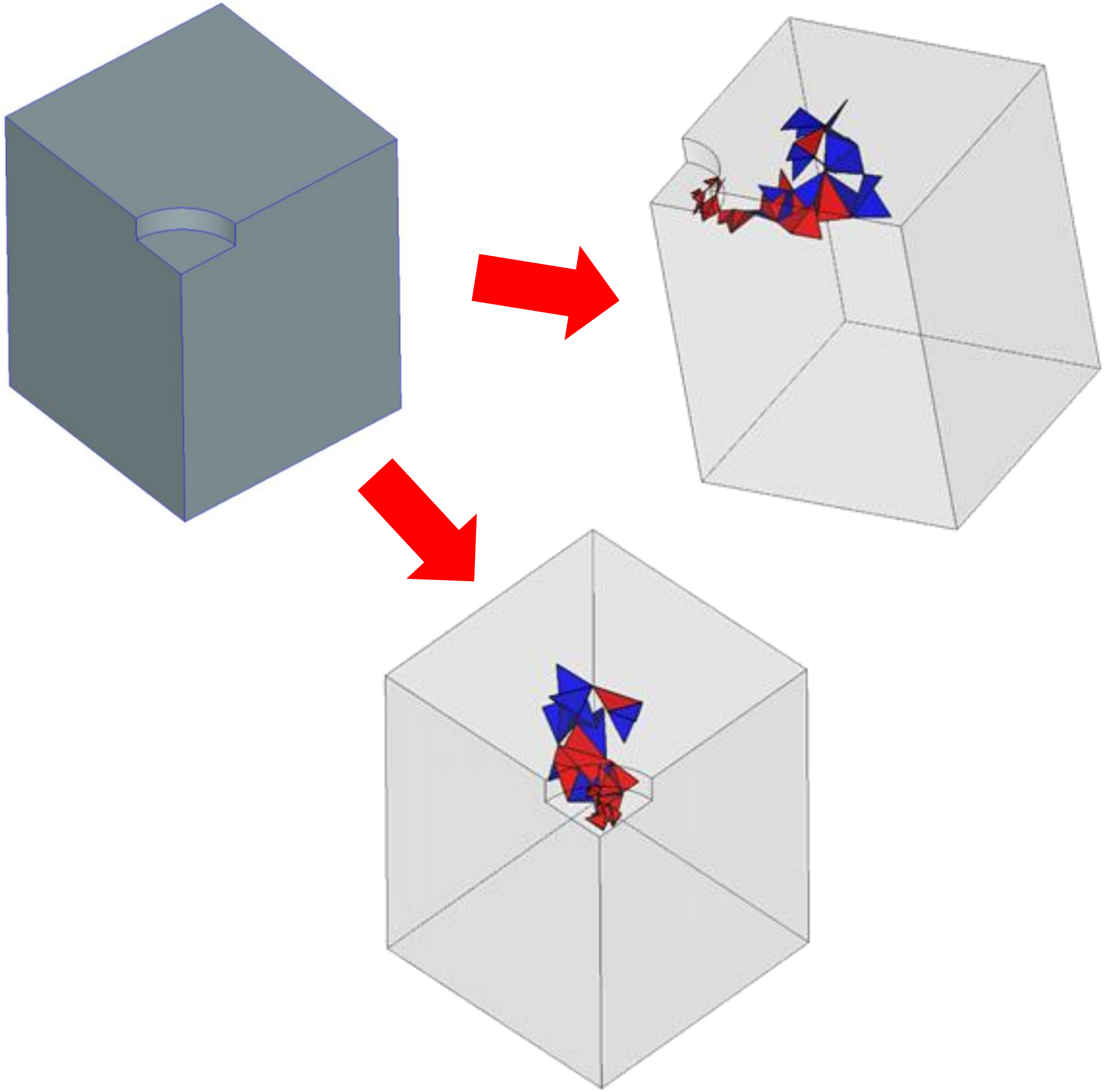
- Boundary conformity
- Smooth partitions

Cons:

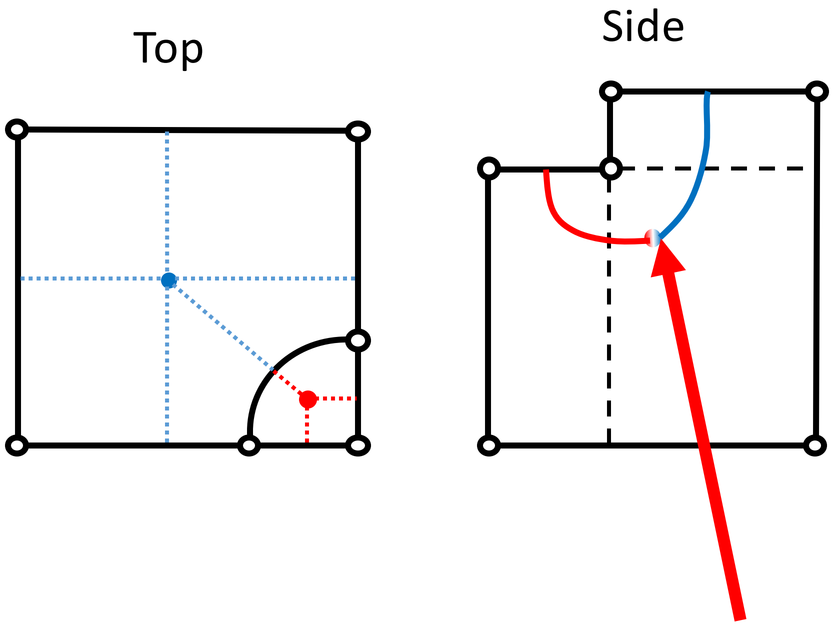
- Correct internal topology is not guaranteed.



Failing example:



Why?



Invalid transition from a negative
to a positive singularity

What about starting from the interior?

- Starting from the boundary may result in an internal singularity topology that is unsuitable for hex-mesh generation.
- What if we created first the singularity network in the interior and then extend it to the boundary?
- But where do we start?
  - A reasonable option is the medial object of the domain.

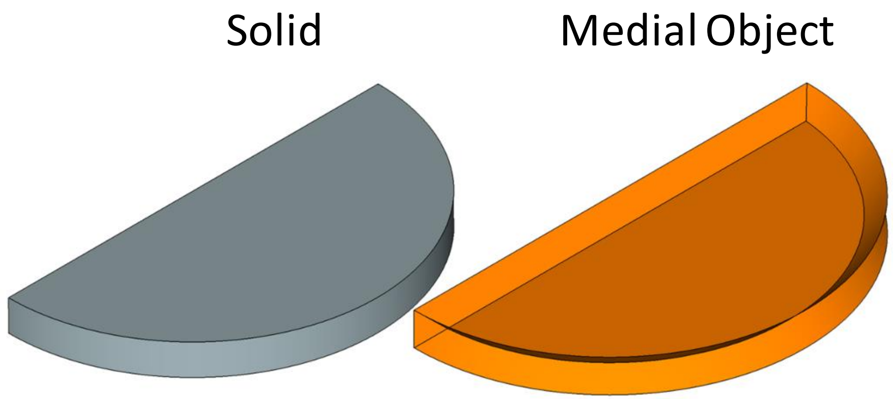
Medial object

- **The medial object can be defined for every 3D domain as**

  The locus of points that are centres of maximal spheres, where a sphere is maximal if it is tangent to the boundary of the domain and it is not enclosed by any other sphere.

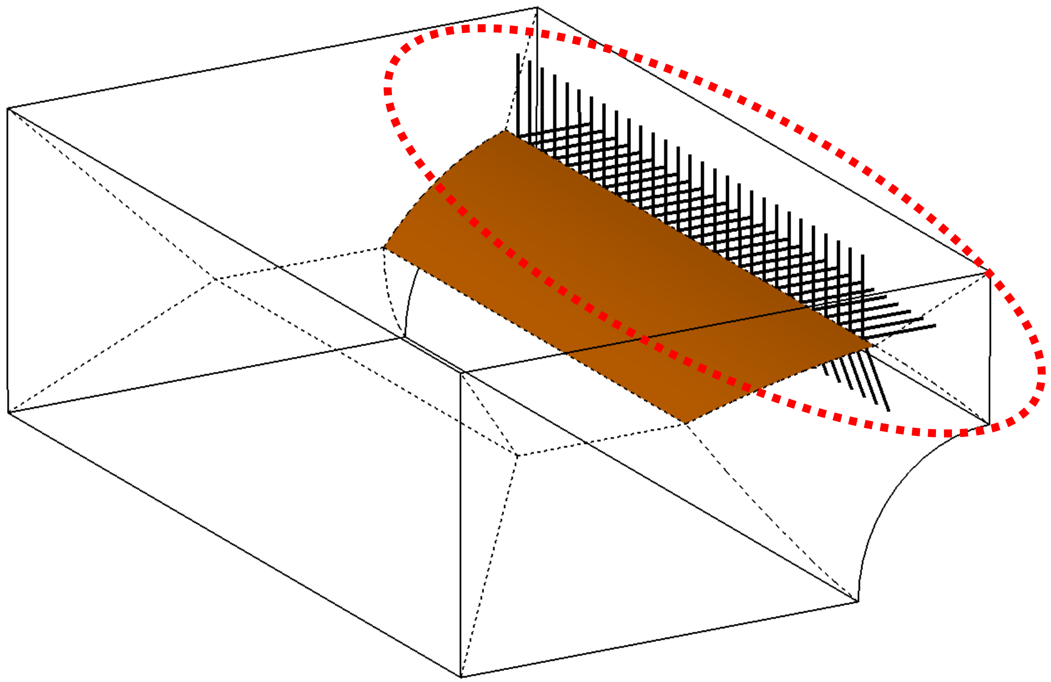
- The medial object has its own structure. It consists of:
  i. Medial Surfaces (2-dimensional)
  ii. Medial Edges (1-Dimensional)
  iii. Medial Vertices (0-Dimensional)

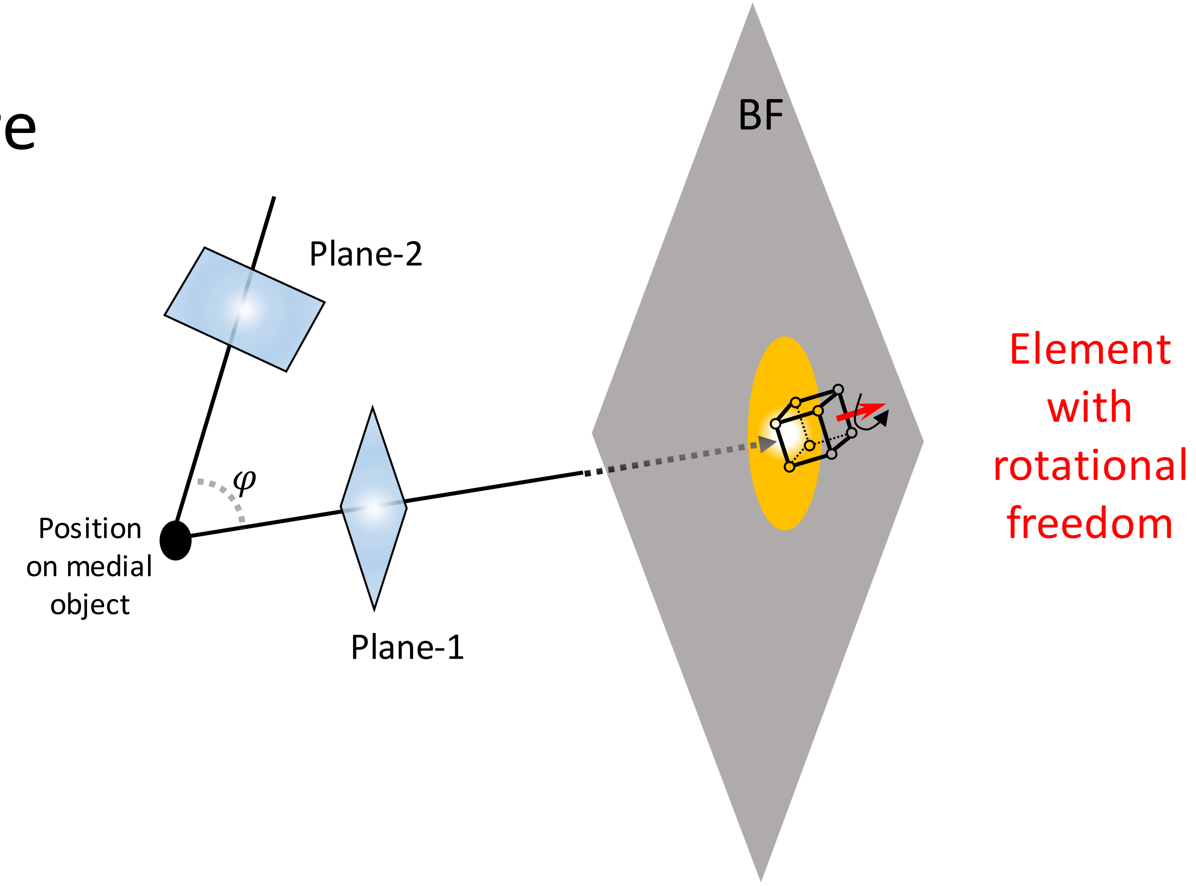
- Important Properties
  i. Dimensional reduction (3D → 2D)
  ii. Unique equivalent representation of geometry
  iii. Orientation independent
  iv. Topology equivalence
  v. Identifies boundary entities in proximity



Medial object (touching vectors)

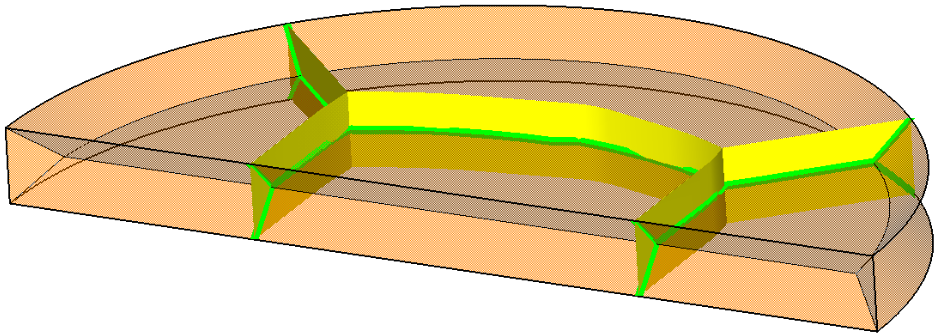
- Connection between the medial object and the boundary
- Normal to the boundary
- Length = radius of inscribed sphere





Intersections with the medial object

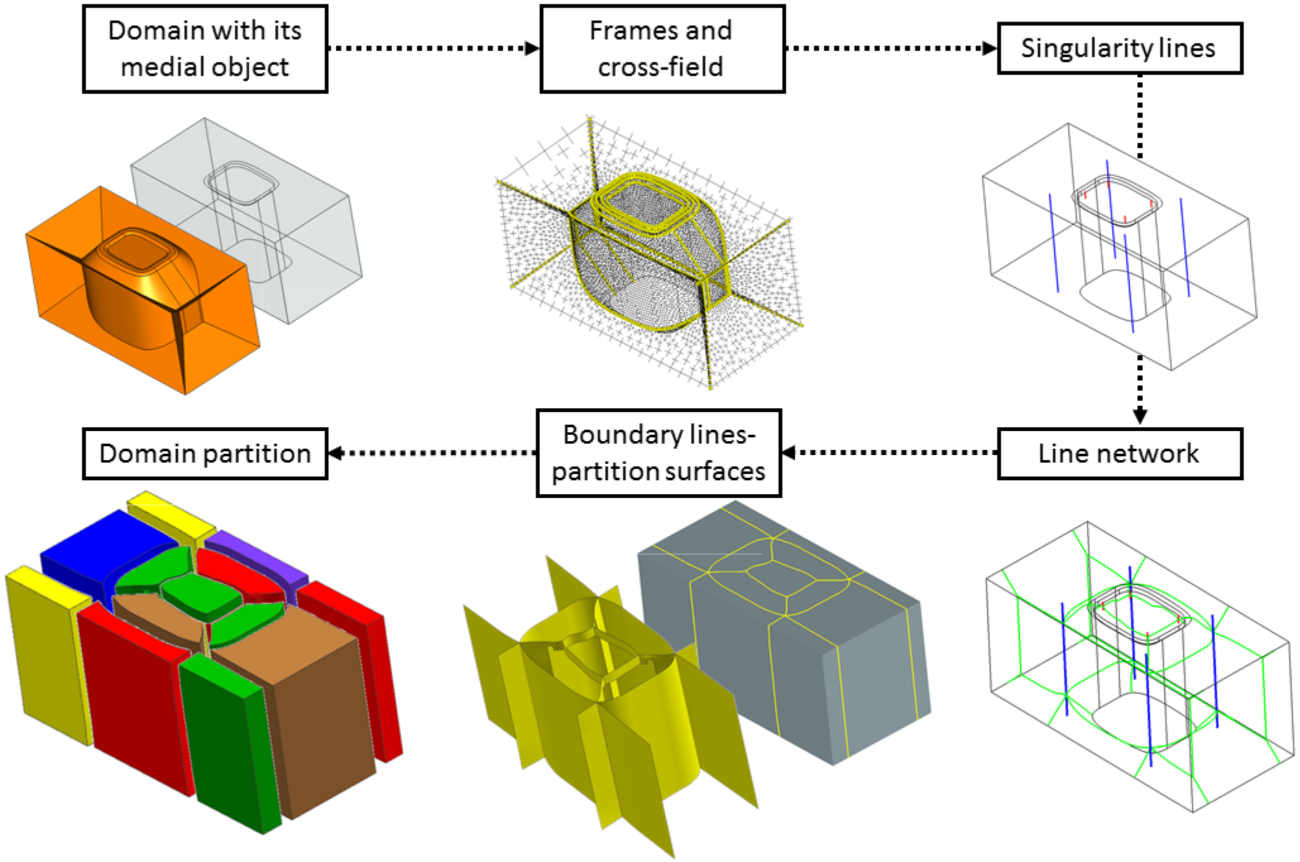
- Lines are defined by the intersection of partition surfaces with the medial object.



Can we identify these lines without previously having the partition surfaces?

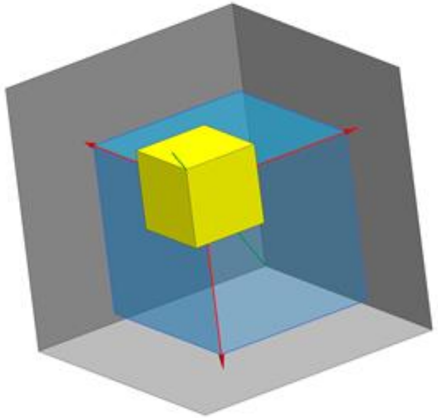
Method

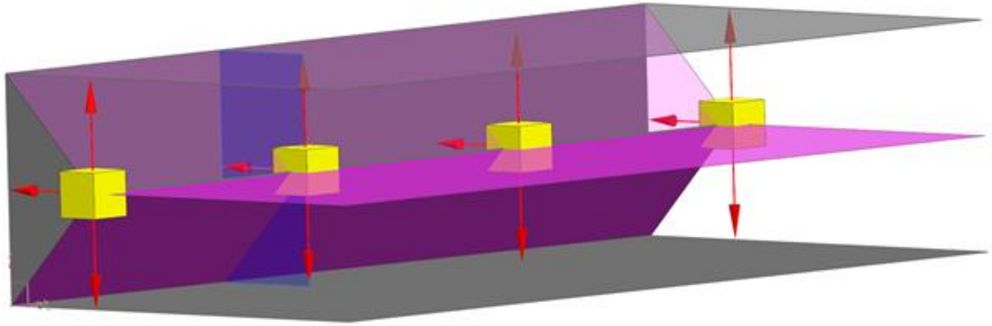
- Try to generate partition surfaces by identifying such lines on the medial object

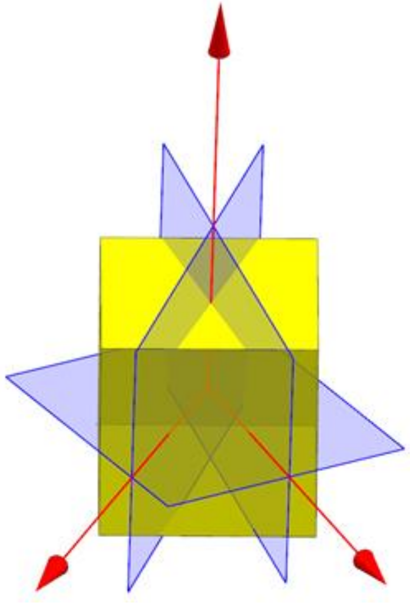


Frames / Cross-fields

- Generate a direction field on the medial object.
- It consists of
  - Frames on medial edges and vertices
  - Cross-fields on medial surfaces
- Frames are generated based on touching vectors

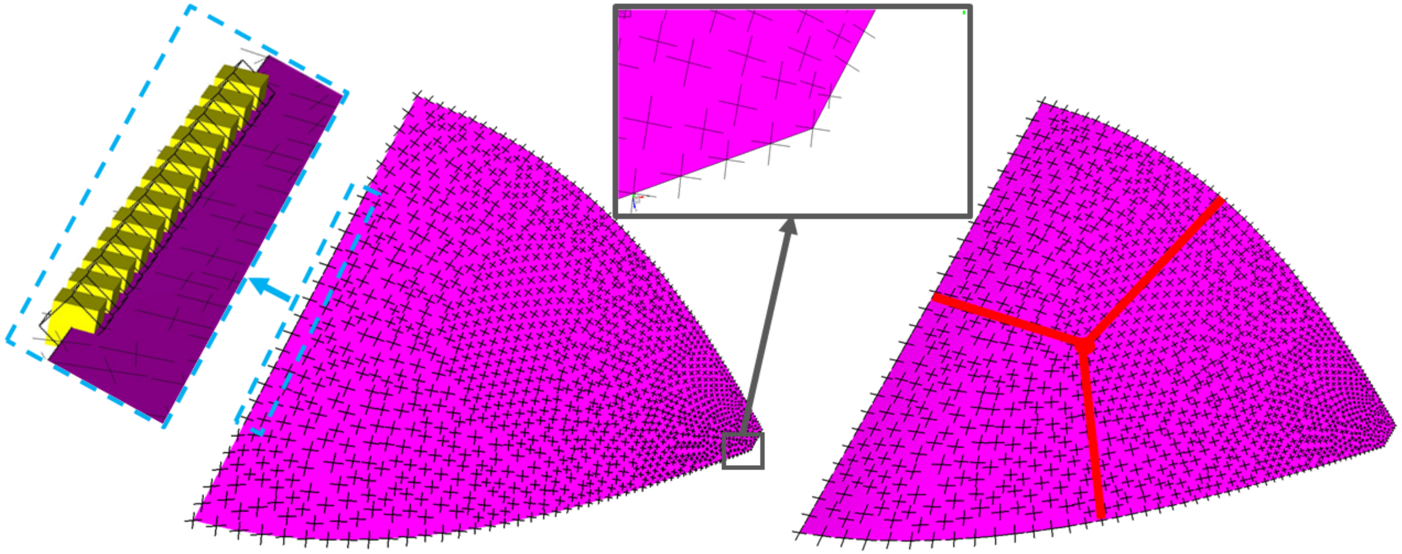






Frames / Cross-fields

- Based on the frames, cross-fields are generated on medial surfaces
  - Propagate crosses and smooth them



Crosses based on the Frames

Crosses aligned with medial edges

- Crosses lie on medial surfaces.
- They are not necessarily tangent to medial edges

Singularities and medial object

Search for two types of singularities:

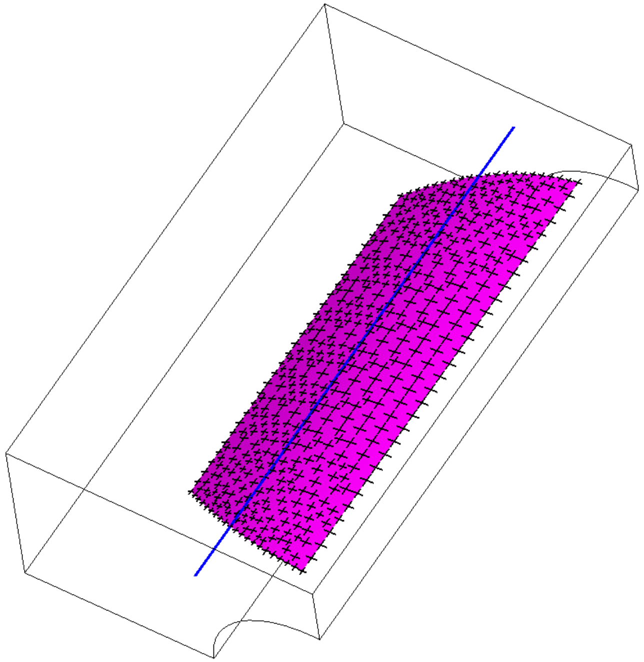
- Type-1:

  Singularities that lie on the medial object.

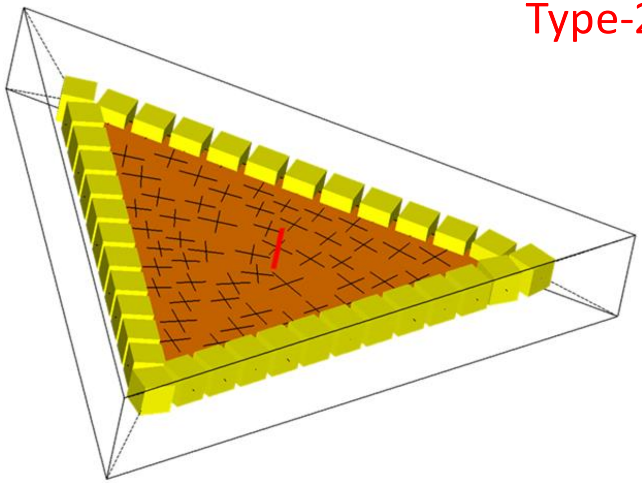
- Type-2:

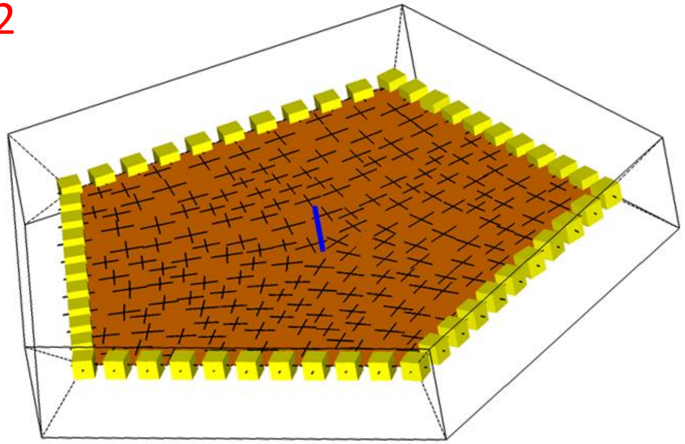
  Singularities that are normal to the medial object (correspond to a singular point on a medial surface).

Type-1



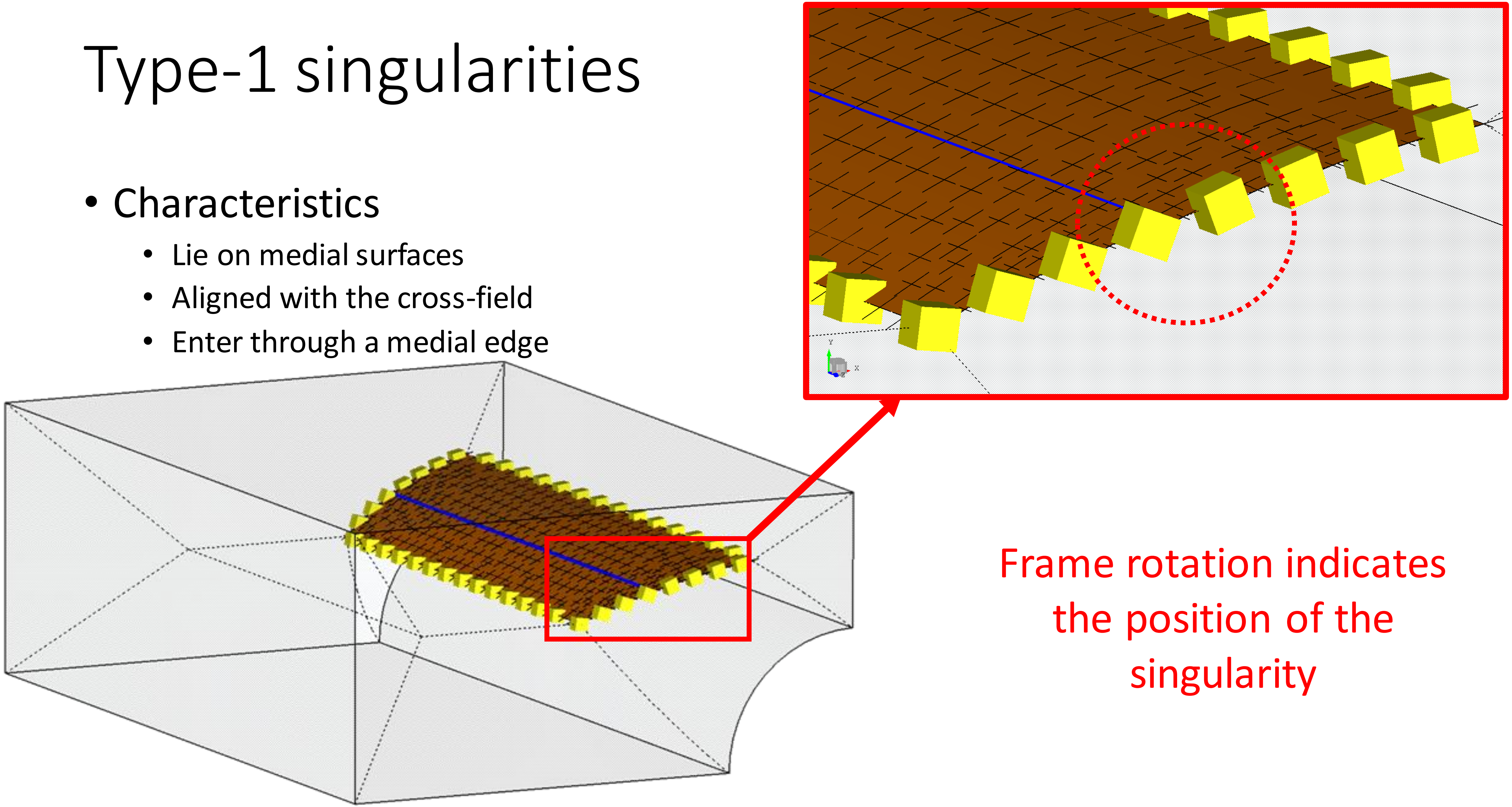
Type-2





Type-1 singularities

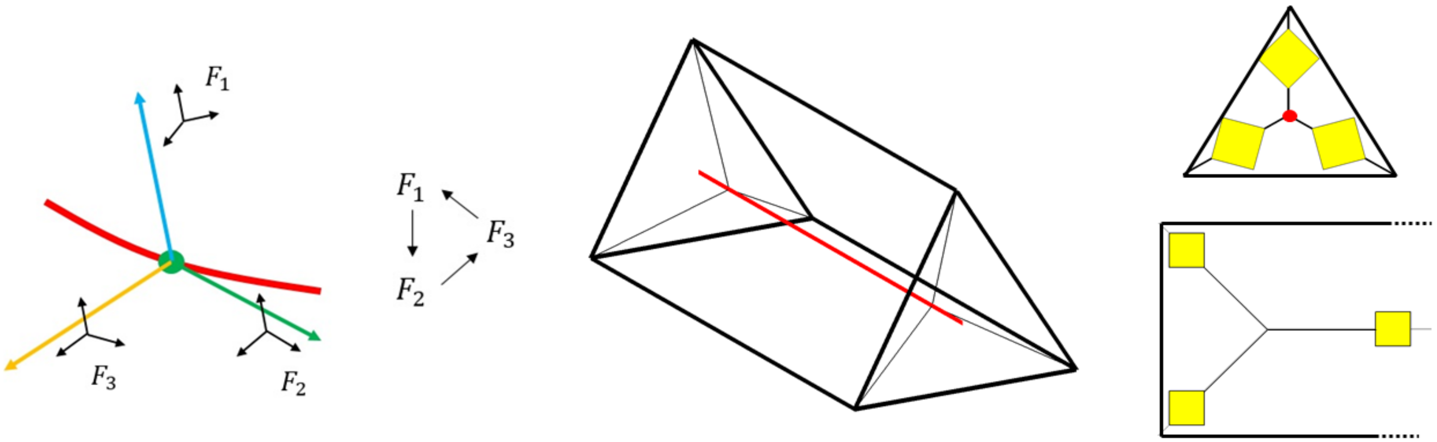
- Characteristics
  - Lie on medial surfaces
  - Aligned with the cross-field
  - Enter through a medial edge



Frame rotation indicates the position of the singularity

Type-1 singularities

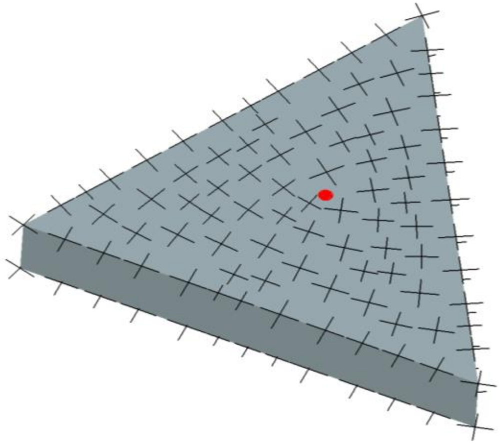
- Singularities can also lie on medial edges
    - Enter through medial vertices

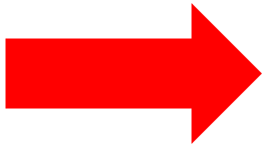


Type-2 singularities

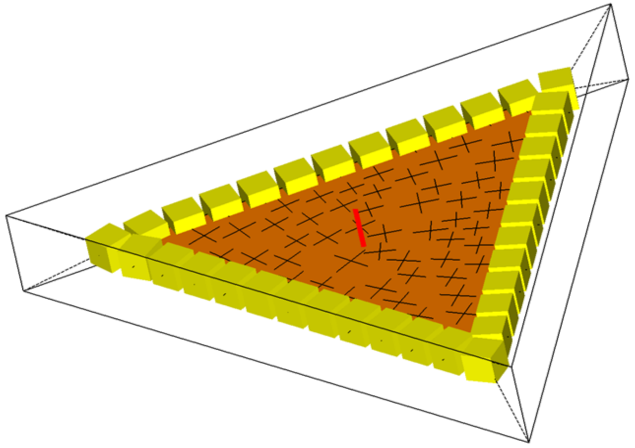
- Analyse cross-fields on medial surfaces
  - Rotations of neighbouring crosses indicate the position and the type of a singularity
  - Extrude to the boundary to construct singularity line

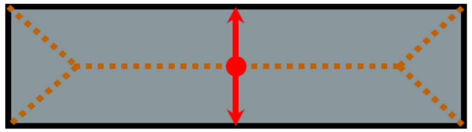
Instead of this





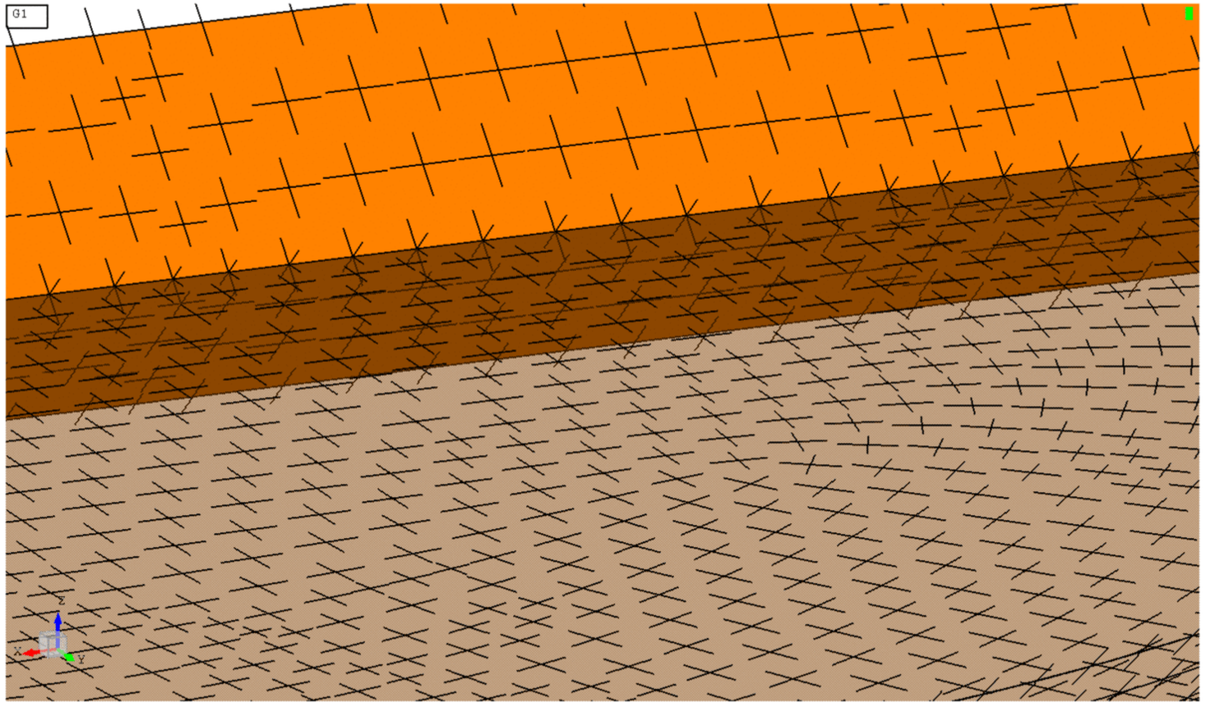
This





Streamlines on the medial object

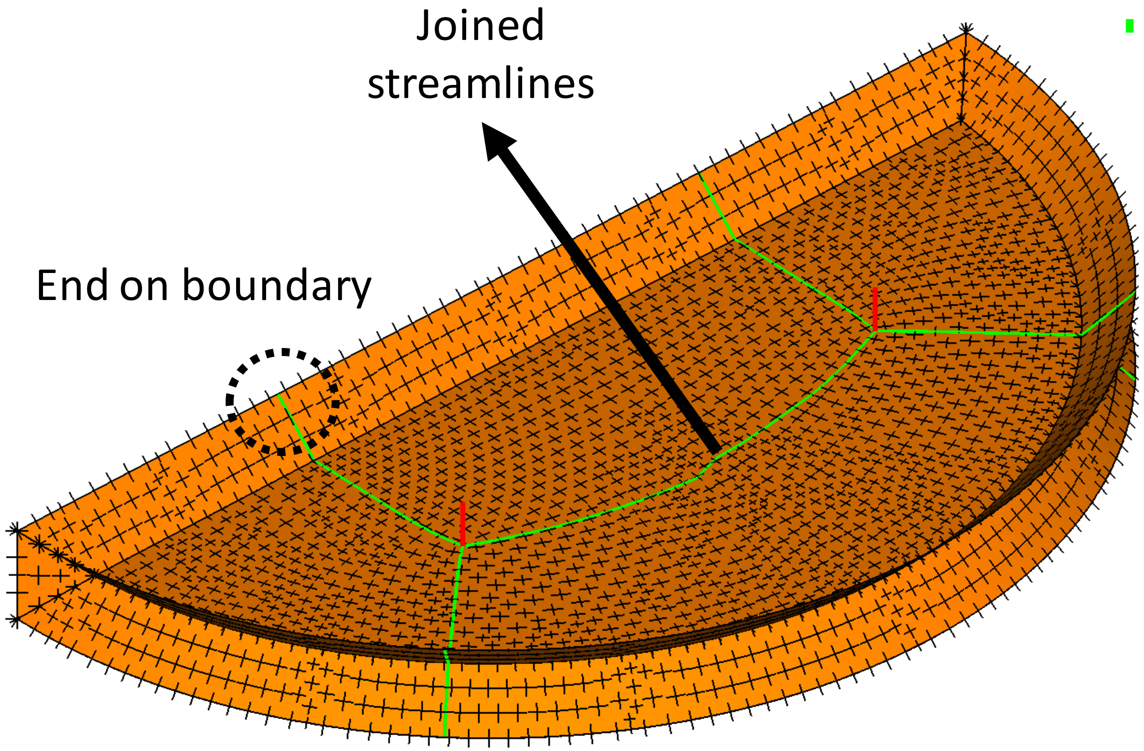
- Like in 2D cross-fields, streamlines emanate from singularities.
  - These are traced on the medial object

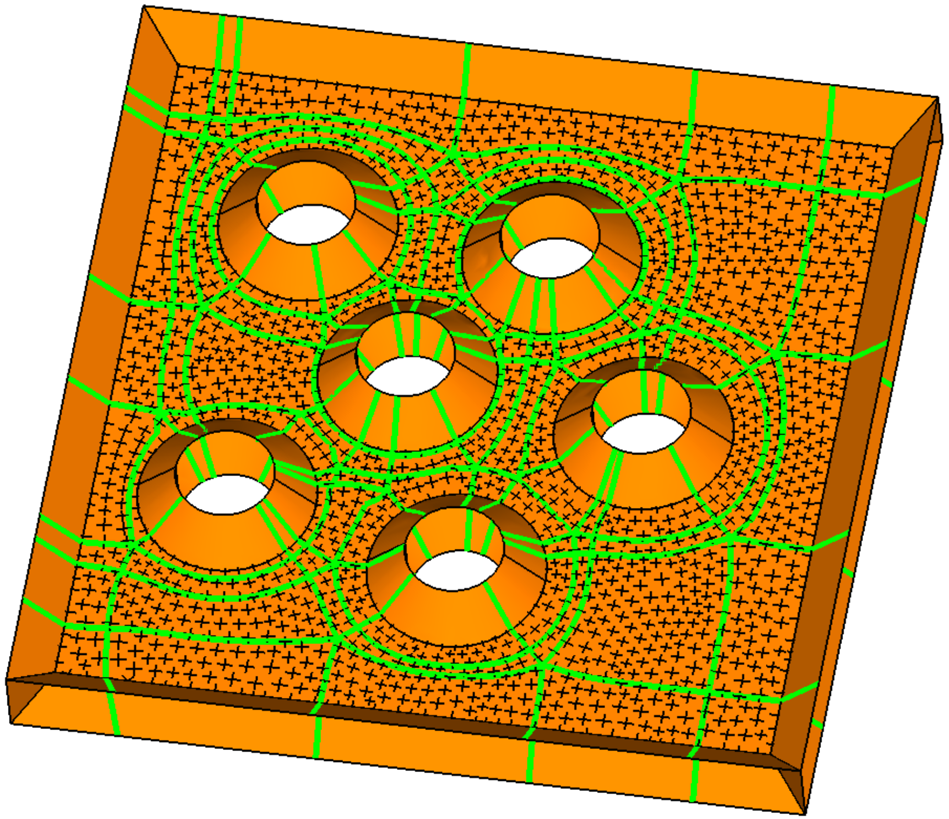


- 3 streamlines (green) emanating from a negative singularity (red)
- On medial edges traces propagate on adjacent medial surfaces.
- Light yellow shows the partition surface implied by the trace.

Streamlines

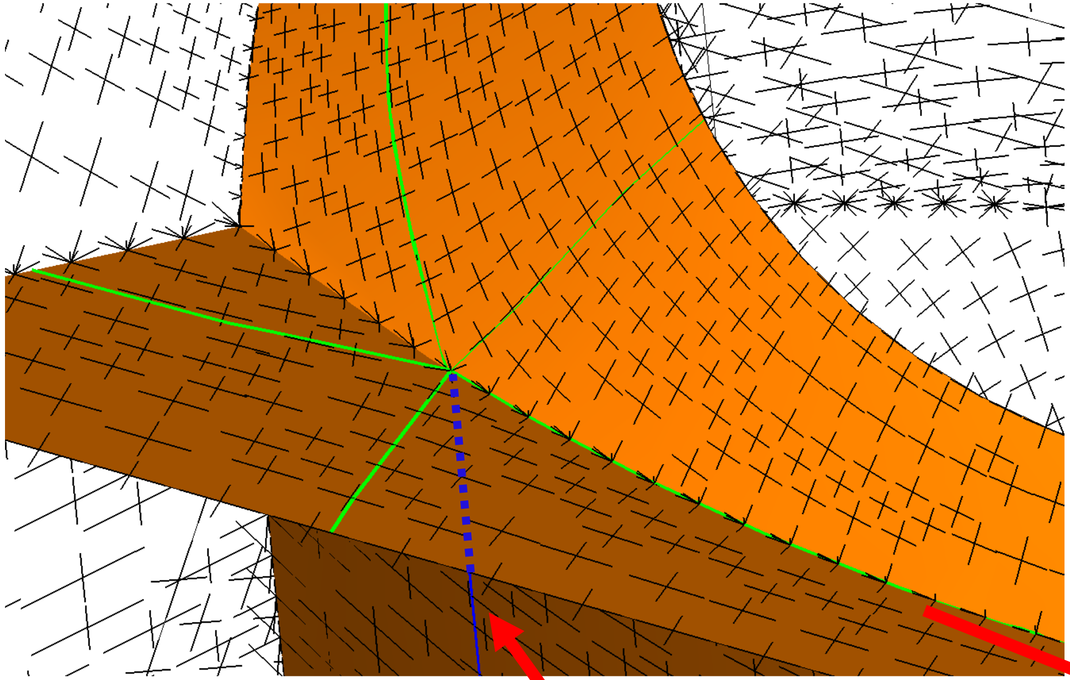
- Streamlines either end on the boundary or join to other singularities.





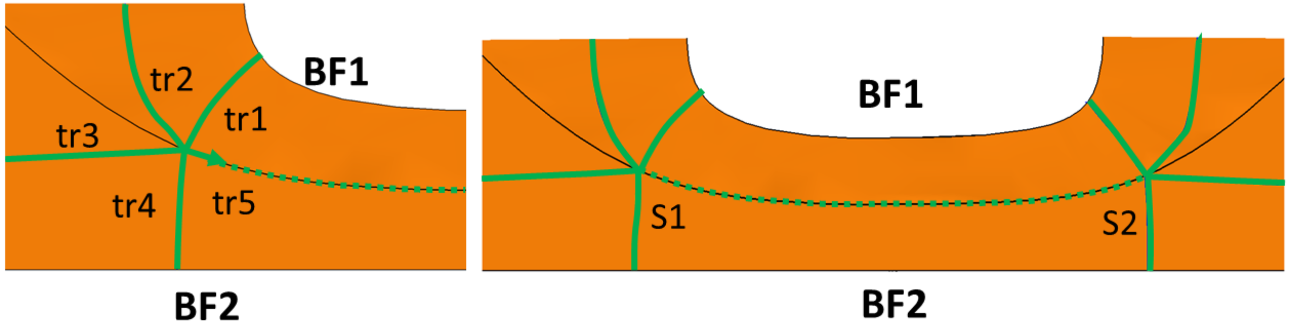
Streamlines

- For Type-1 singularities, streamlines emanate from both ends



Streamlines on one end of a positive singularity

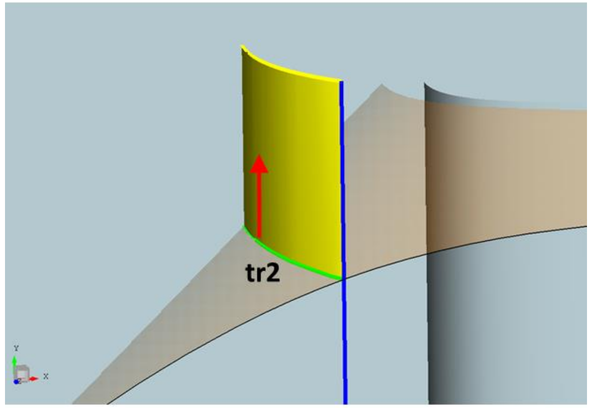
Singularities connected

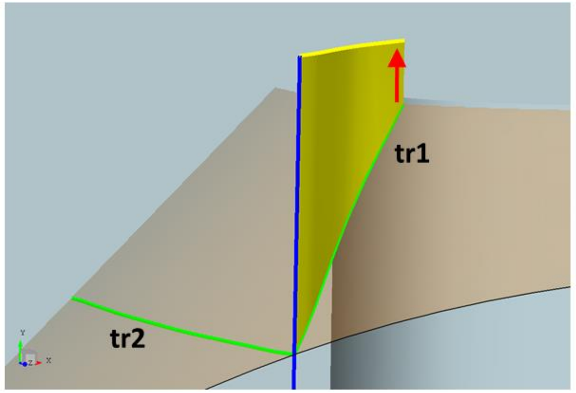


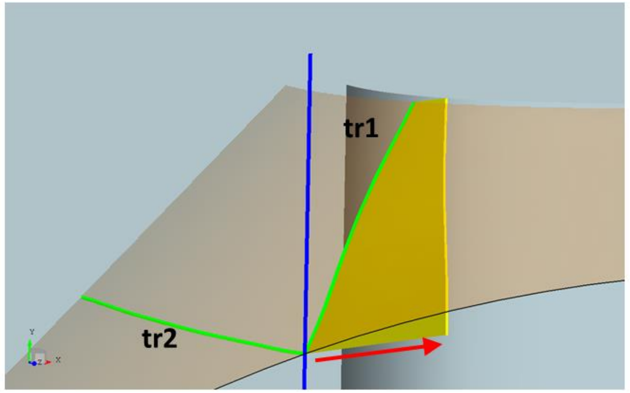
One trace following a medial edge

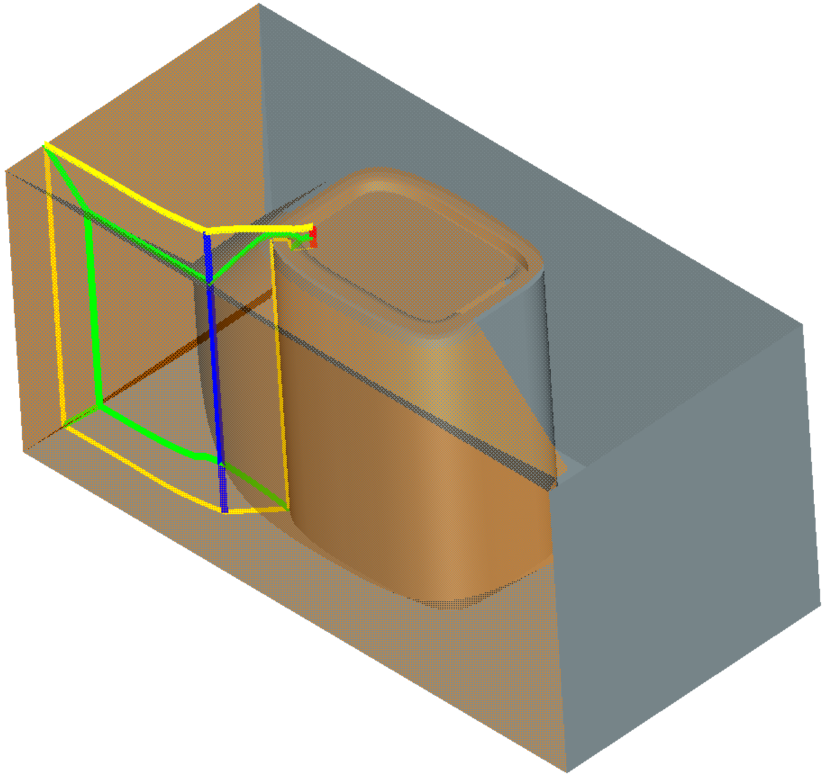
Boundary lines

- Extrude streamlines to the boundary to form boundary loops (yellow).



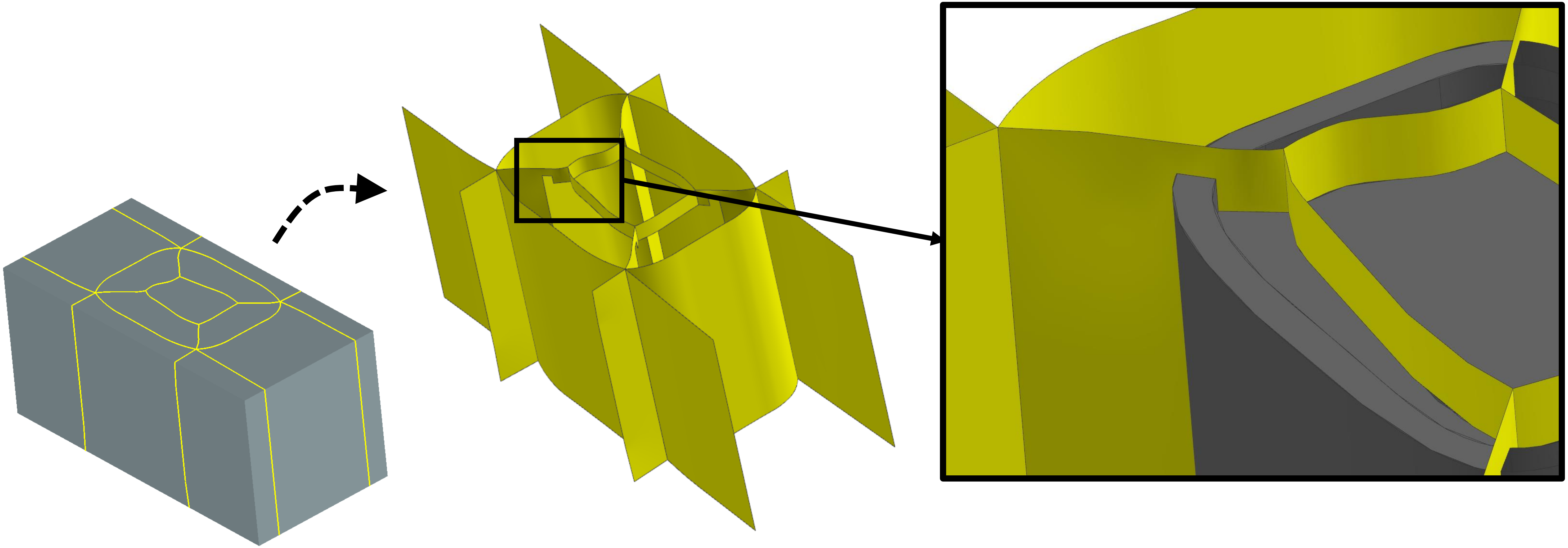




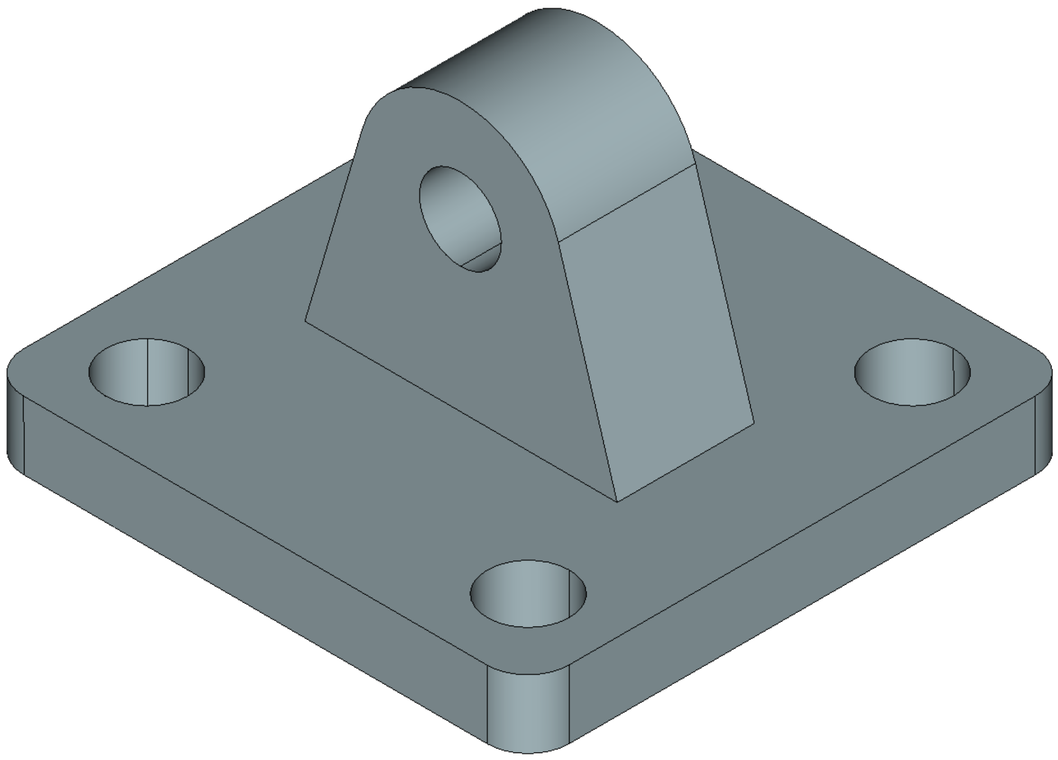


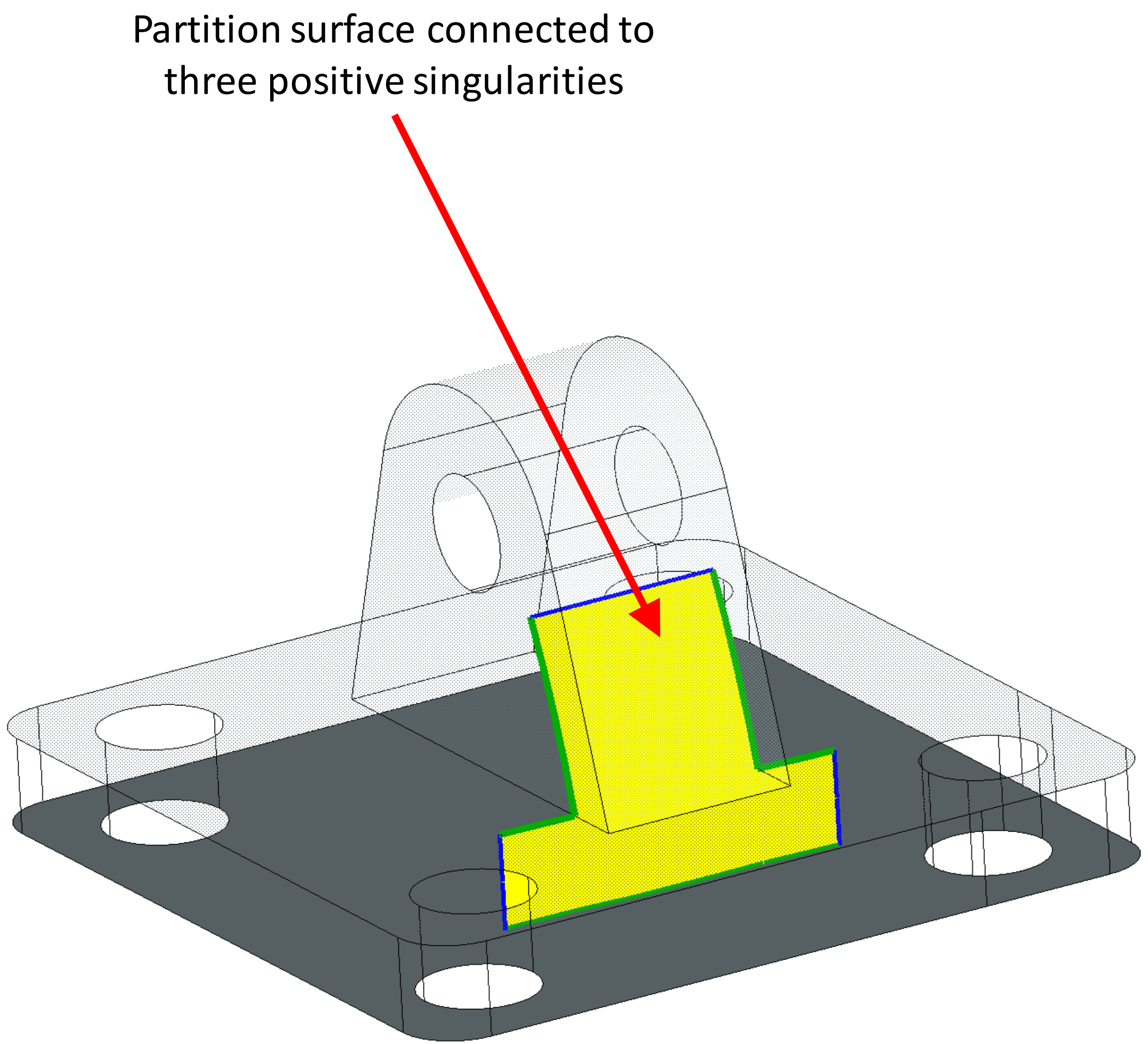
Partition surfaces

- From the loops on the boundary, partition surfaces are created.
- Partition surfaces capture important features of the domain.



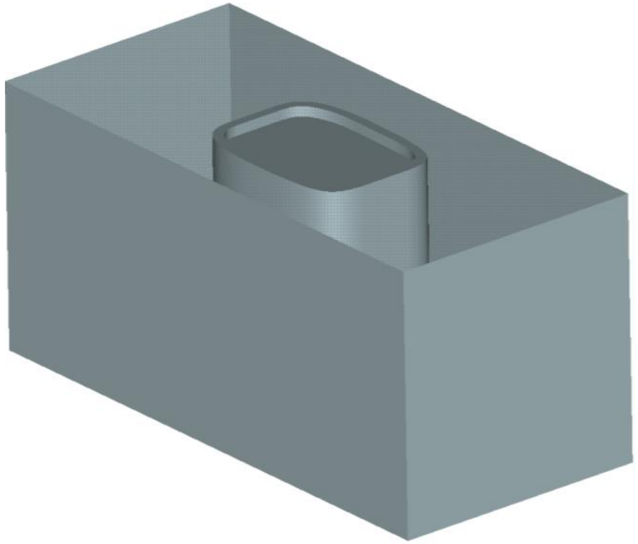
Partition surfaces

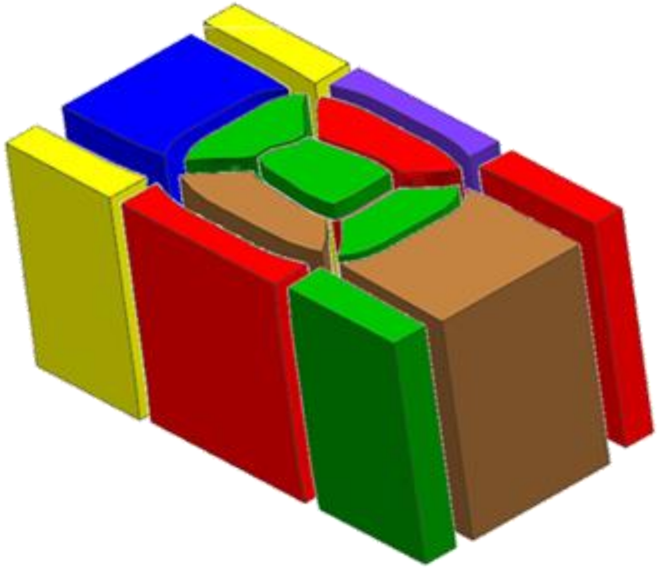


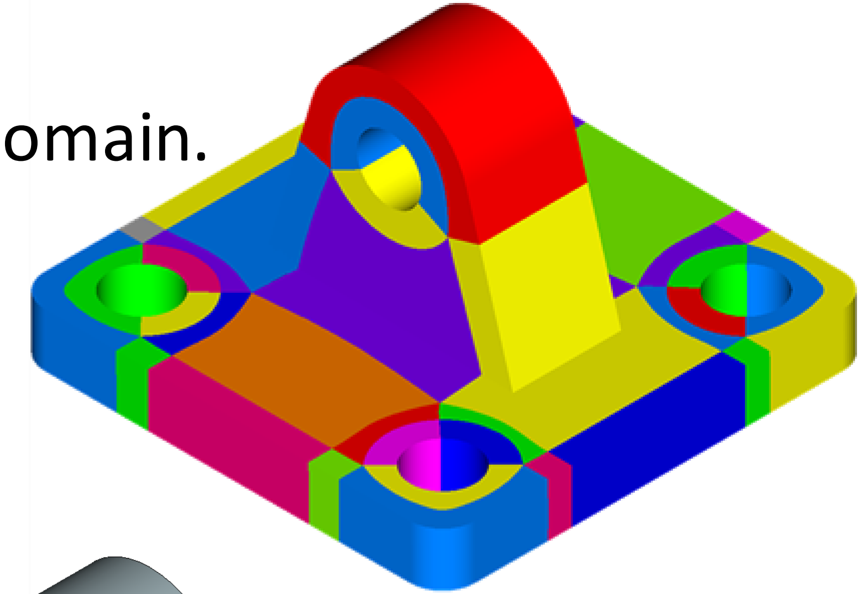


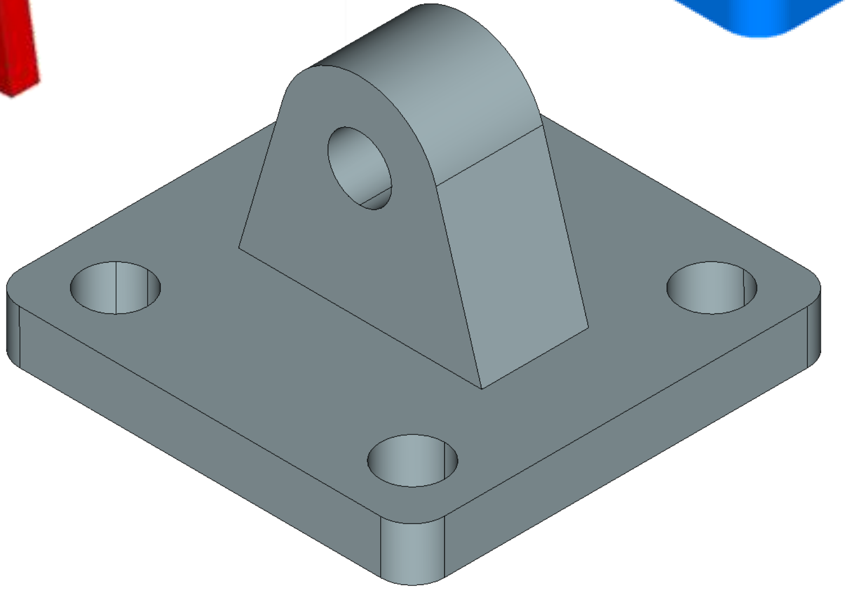
Decomposition

- Partition surfaces are used to decompose the domain.

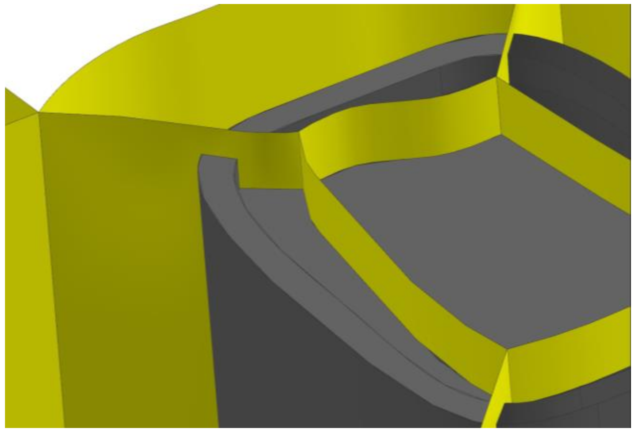




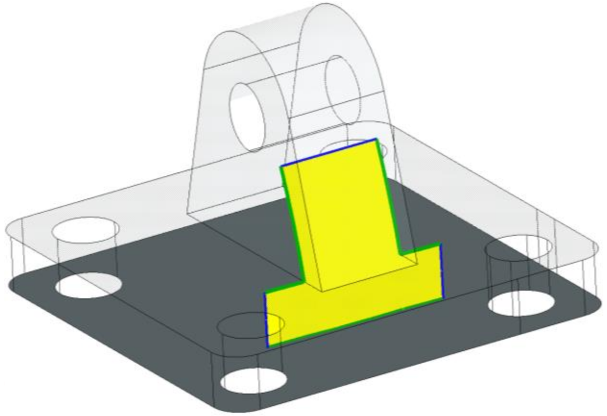


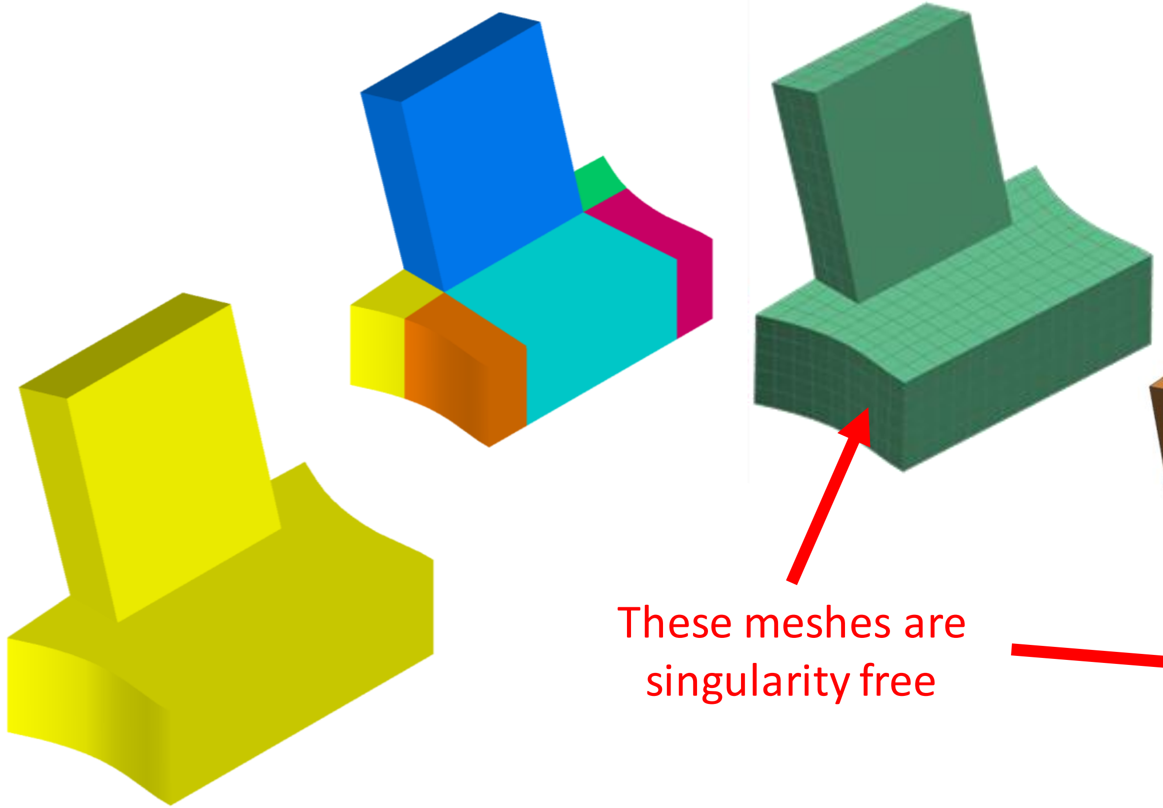


Limitations

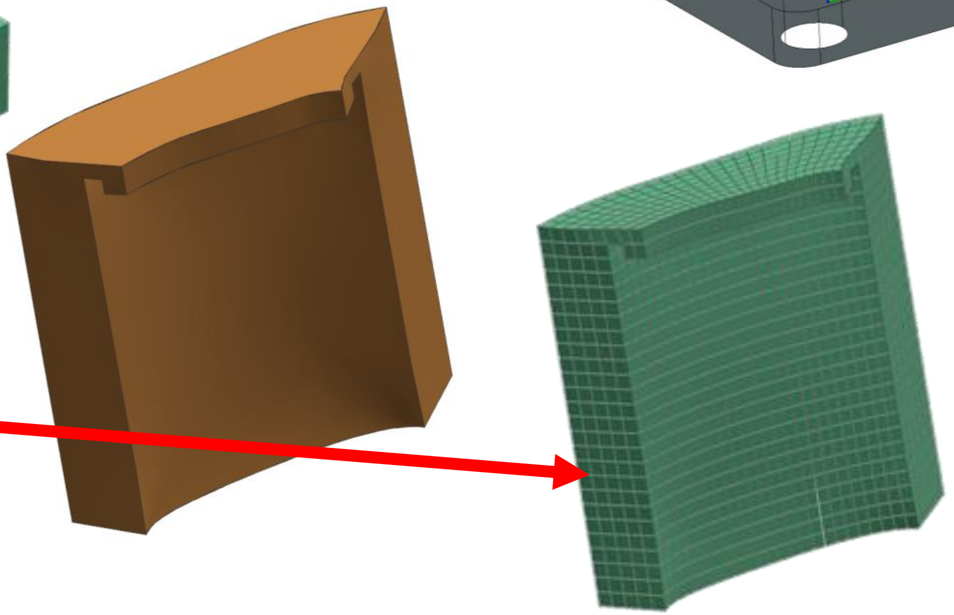


- Concavities:
  - Partition surfaces take into account concavities
  - But regions that are not simple blocks emerge

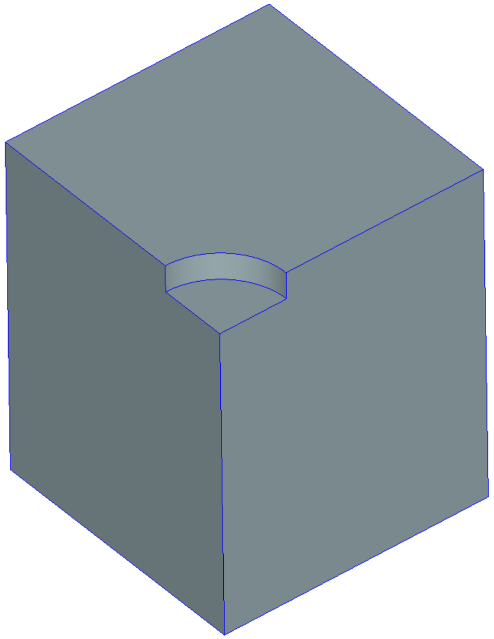


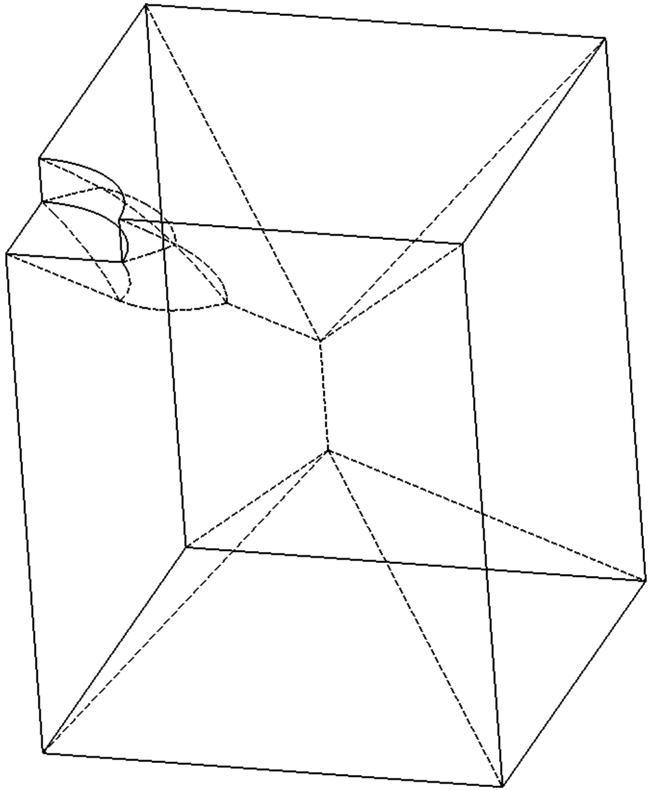


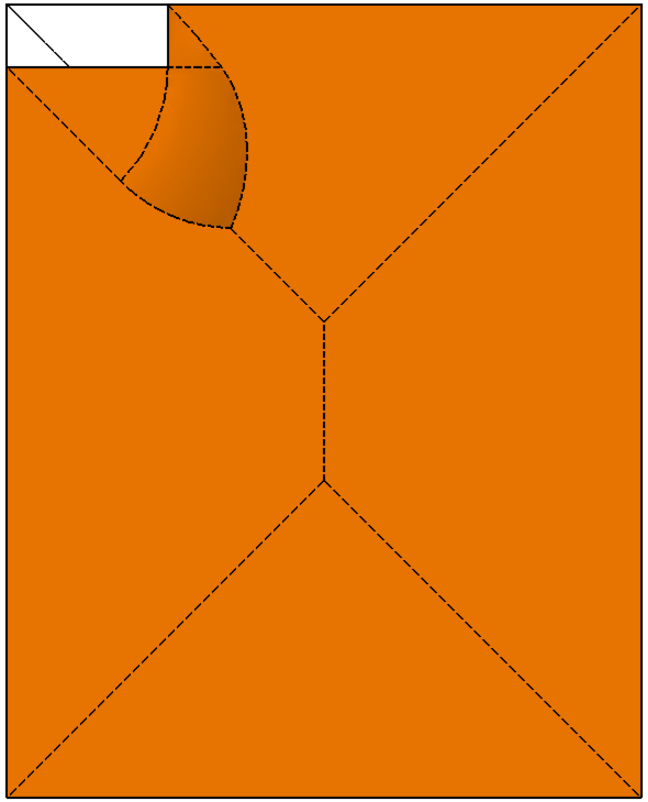
These meshes are singularity free



Limitations



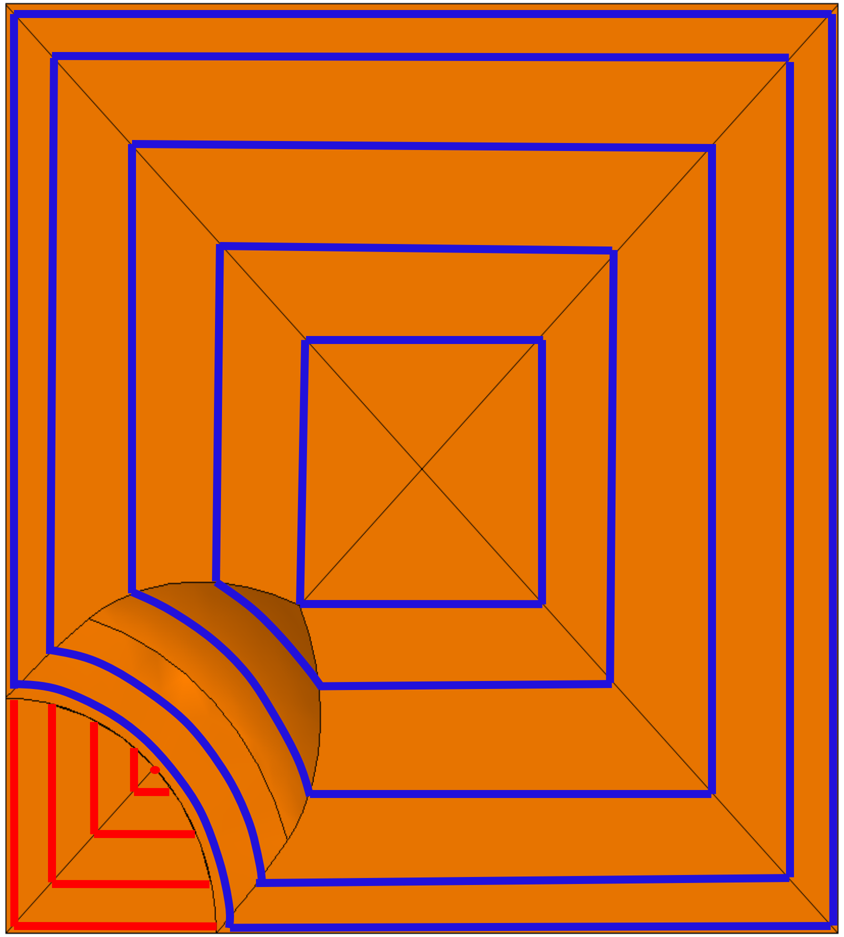


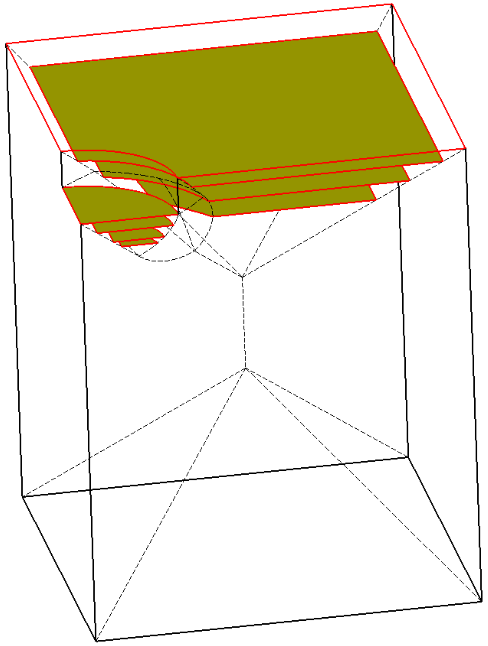


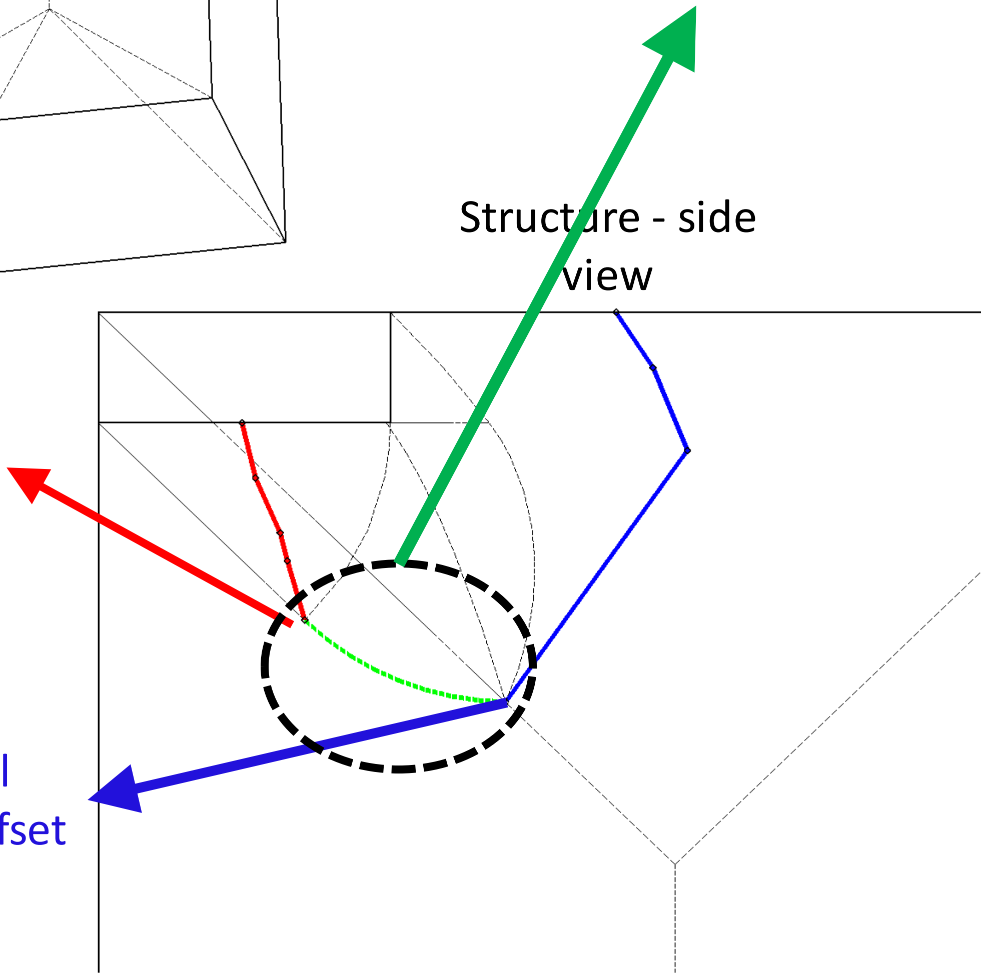
- Isolated concavity
- Long object

Singularity structure

Top view of the MO

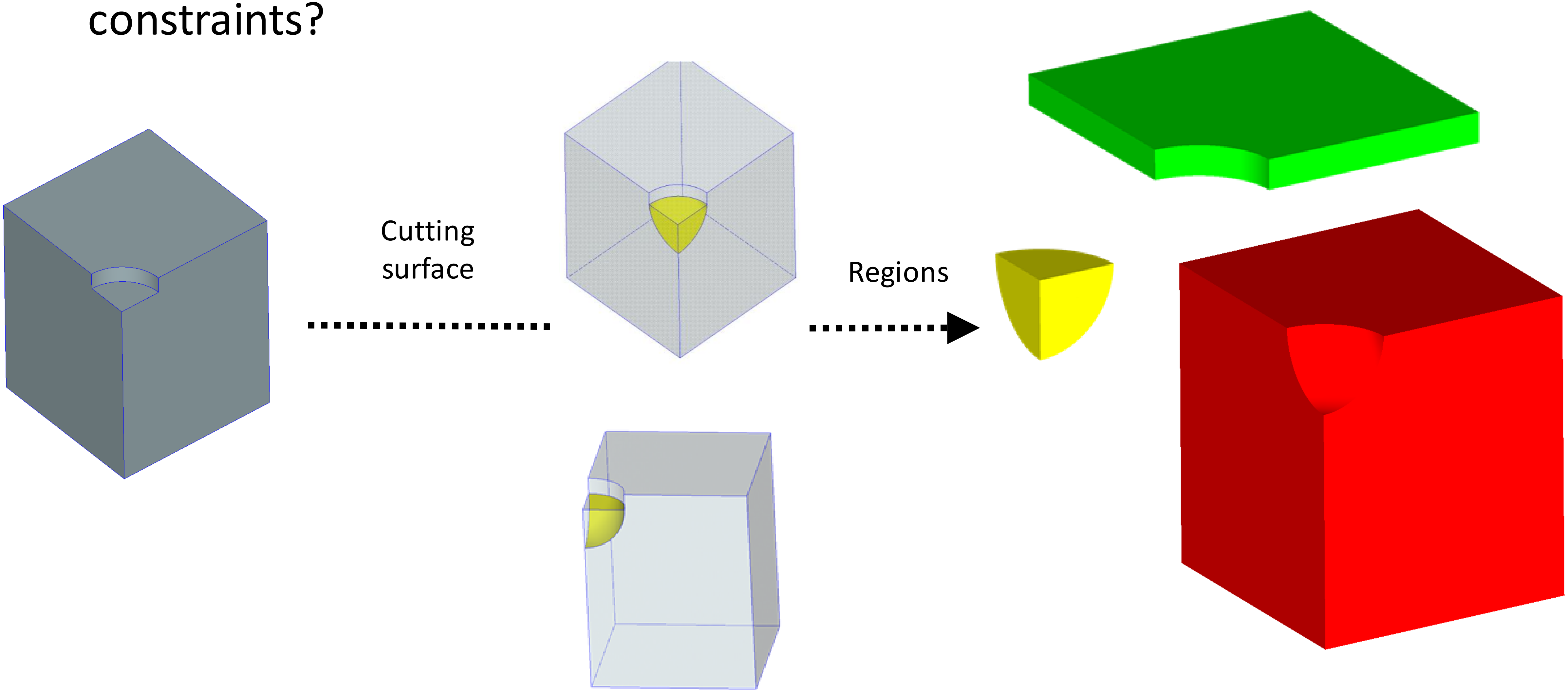






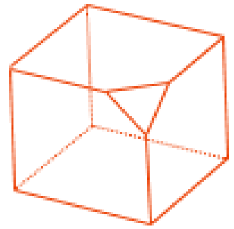
Limitations

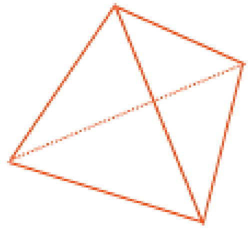
- Could we make use of the structure to impose internal constraints?



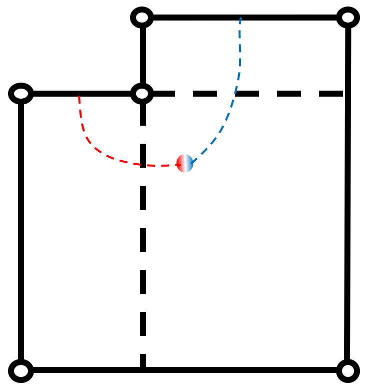
Limitations

- Now the topology of the singularities is forced to change.
- The negative singularity breaks into three negative
- The positive into two positive and one negative

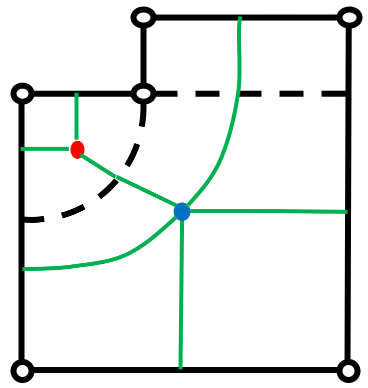




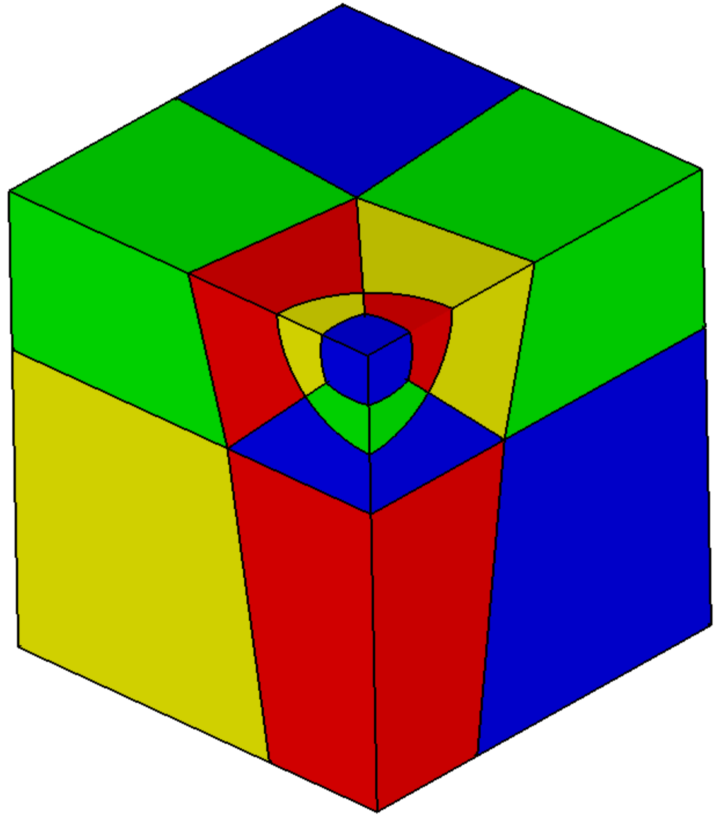
Side



Side



Blocking



The net sum of singularities on the boundary remains 0 !!!

Conclusions

- Singularities have a strong relation to the medial object.
- A strategy is proposed to search for singularities directly on the medial object.
- Based on streamlines emanating from singularities, partition surfaces can be constructed.
- Singularities are pushed far from the boundary.
- Provides a structure suitable for imposing internal constraints.
- Tool to investigate the interior of the object.