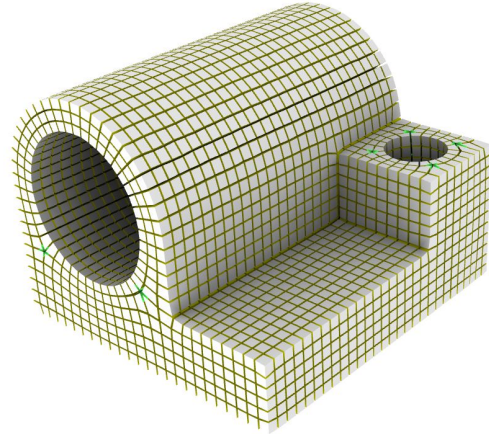
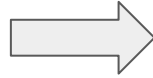
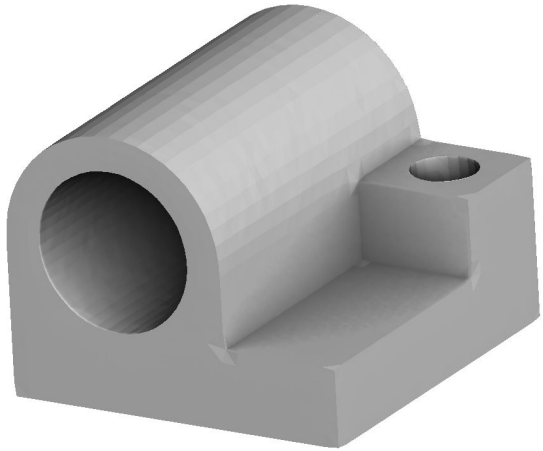


On the bijectivity of polycube maps

Finding a good grid parameterization

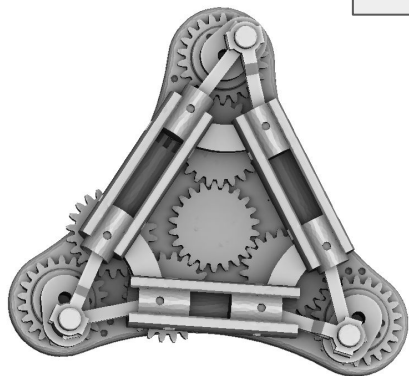
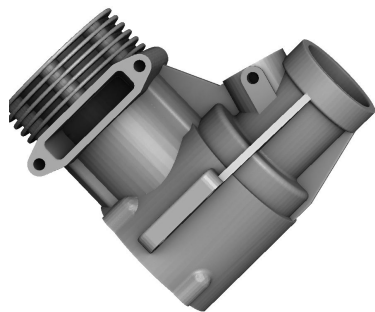
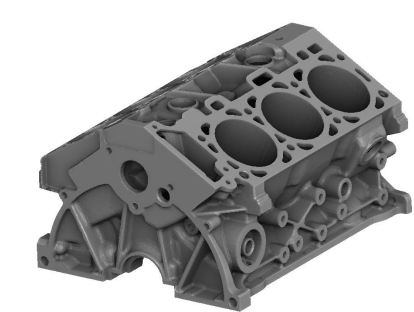
1. Introduction: Hexahedral meshing

Principle



1. Introduction: Hexahedral meshing

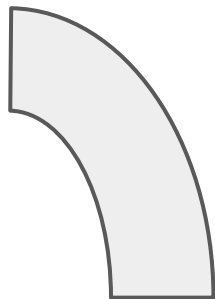
Our dream



??

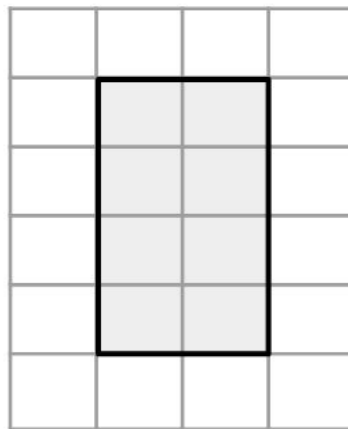
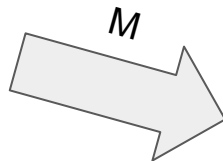
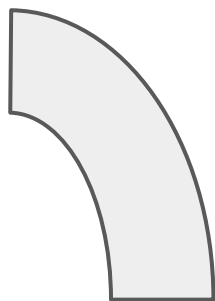
1. Introduction: Global parameterization

Basic introduction

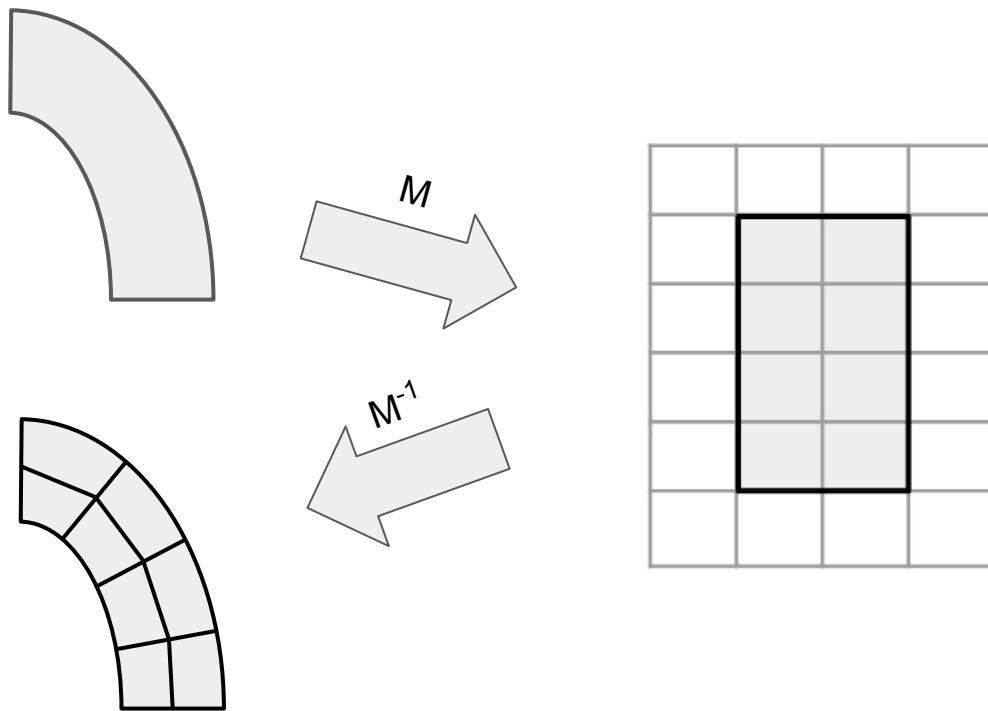


1. Introduction: Global parameterization

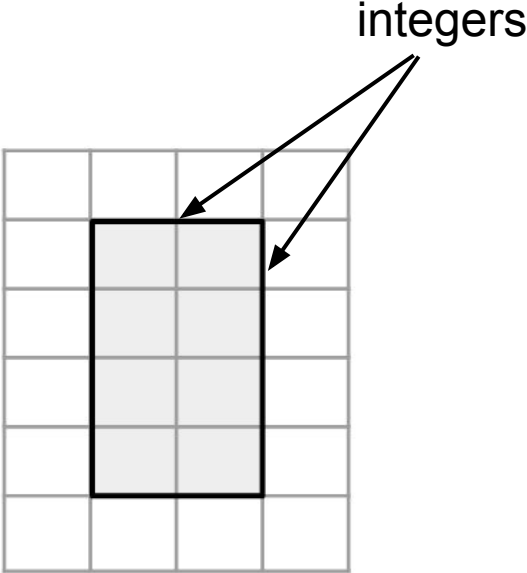
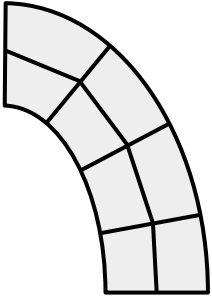
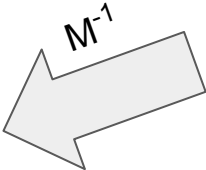
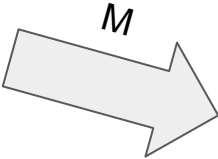
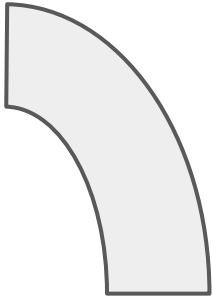
Basic introduction



Basic introduction

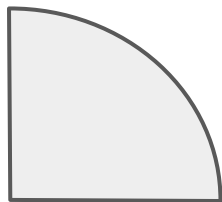


Basic introduction



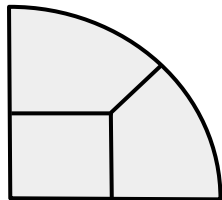
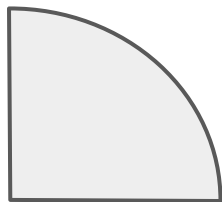
1. Introduction: Global parameterization

More complicated domains



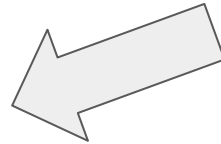
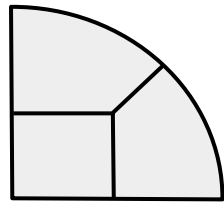
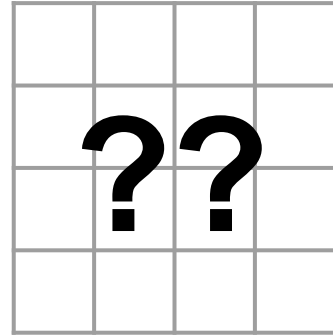
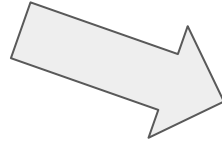
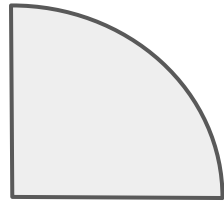
1. Introduction: Global parameterization

More complicated domains

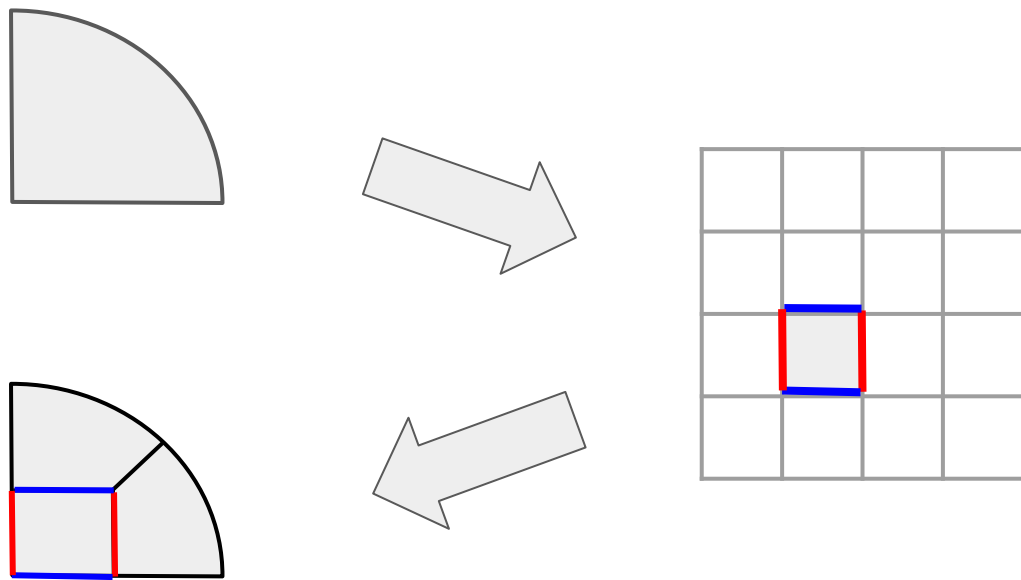


1. Introduction: Global parameterization

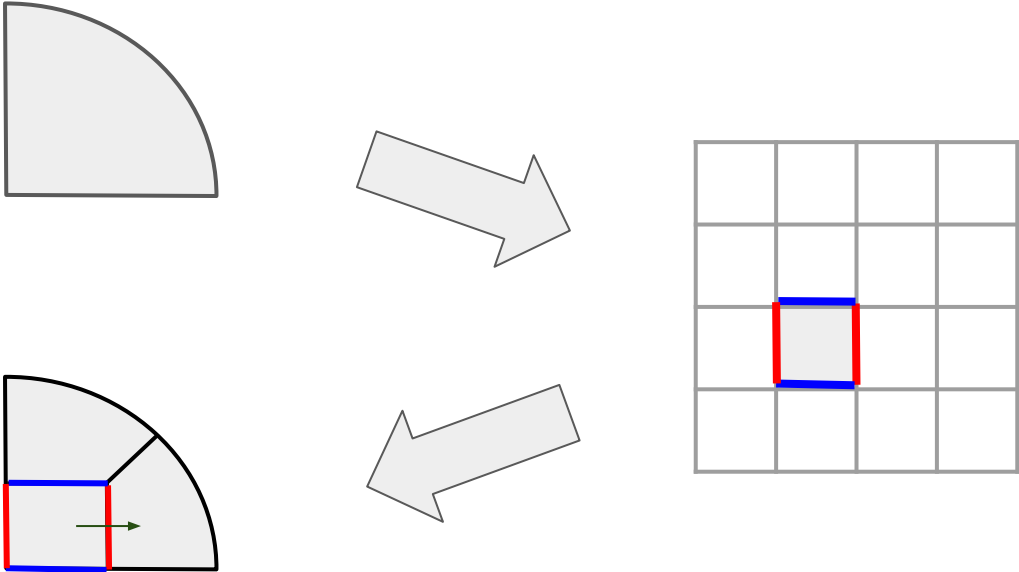
More complicated domains



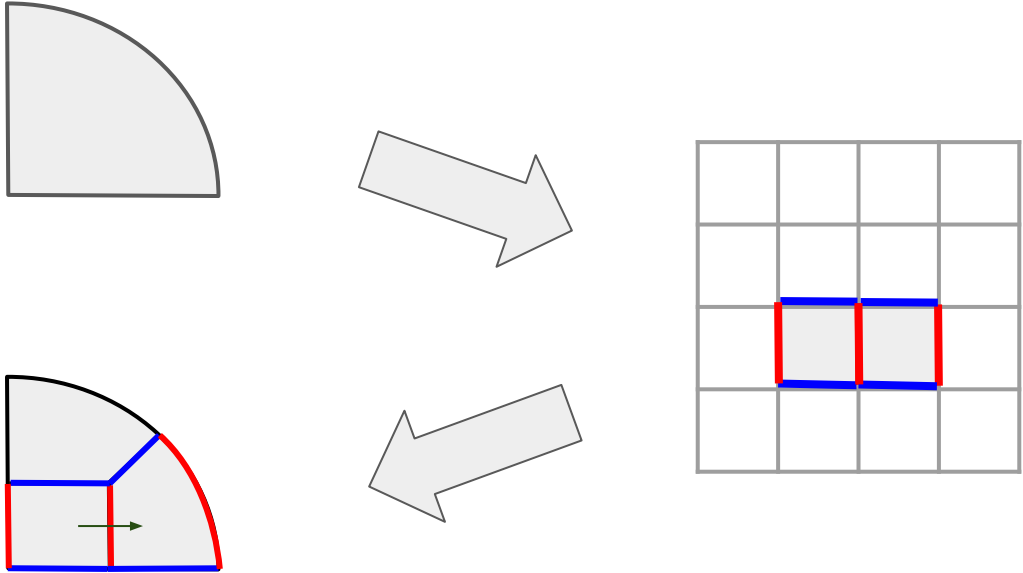
More complicated domains



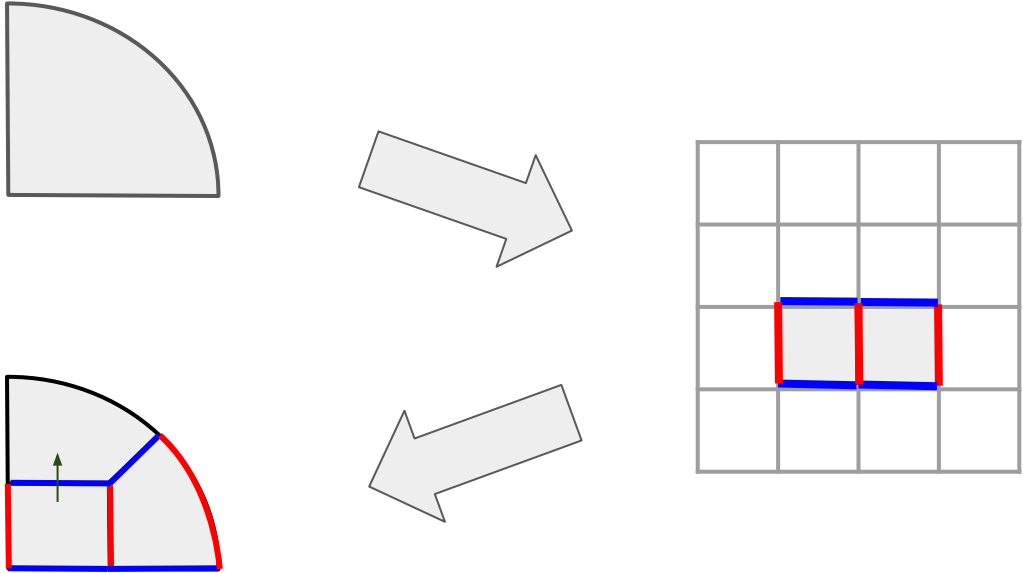
More complicated domains



More complicated domains

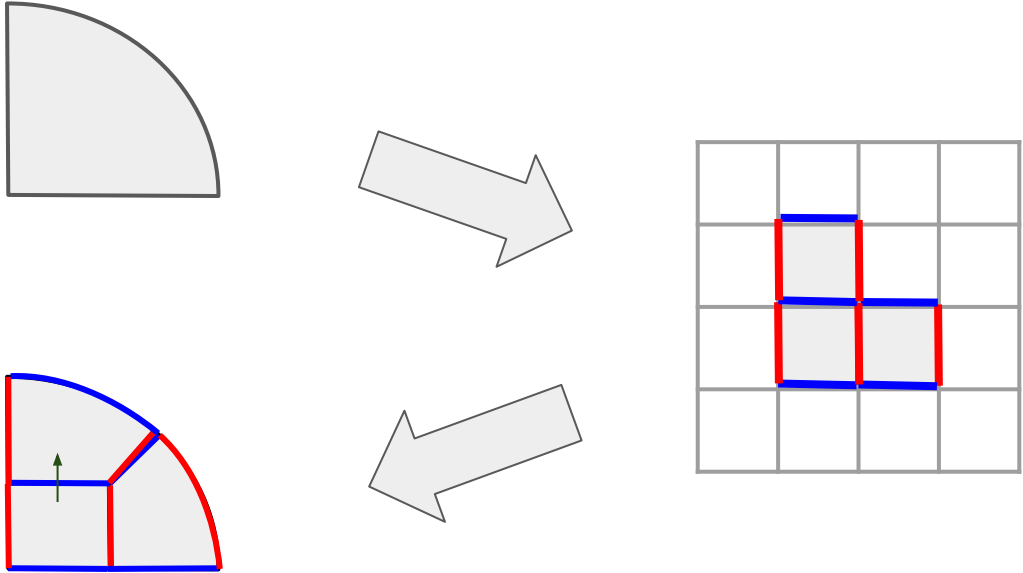


More complicated domains

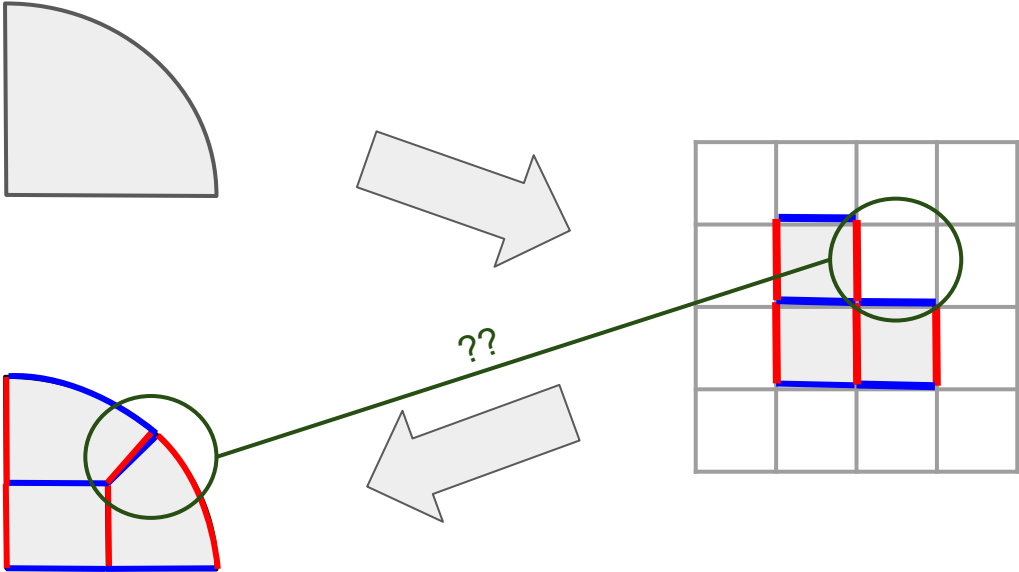


1. Introduction: Global parameterization

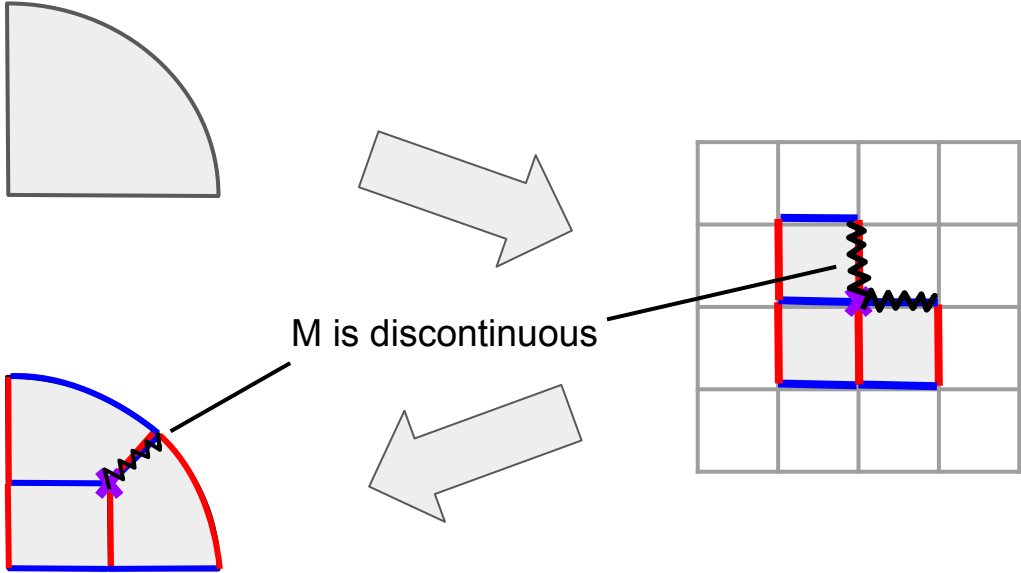
More complicated domains



More complicated domains

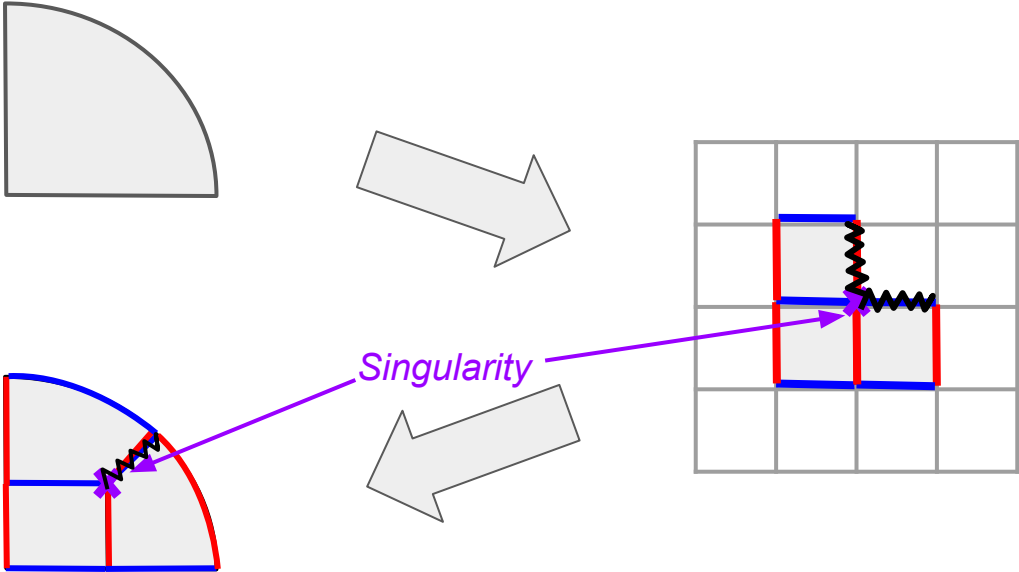


More complicated domains



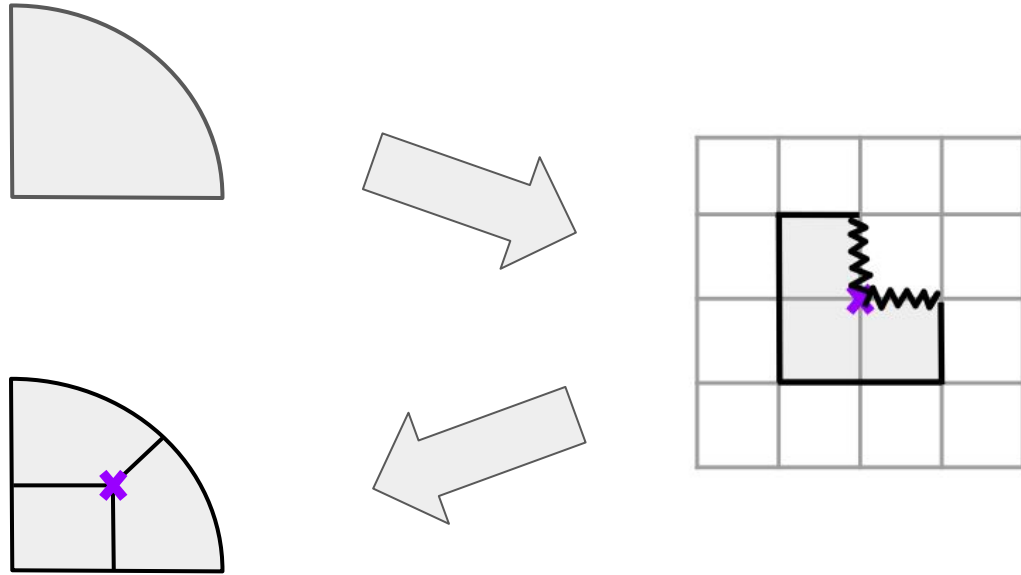
1. Introduction: Global parameterization

More complicated domains

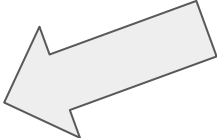
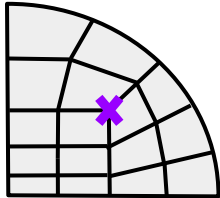
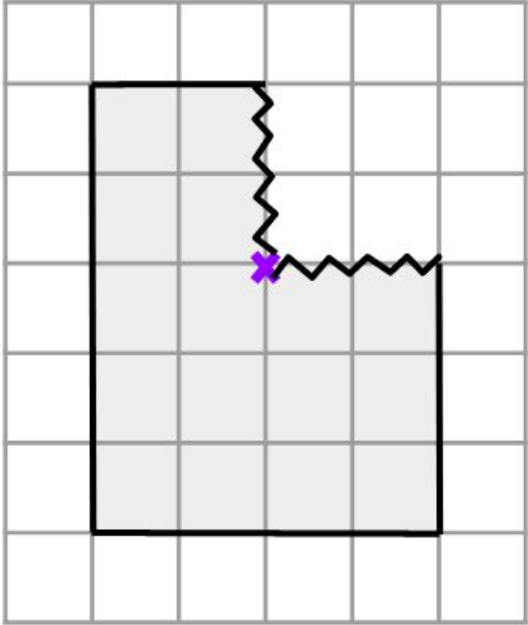
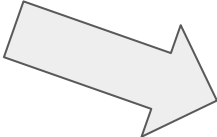
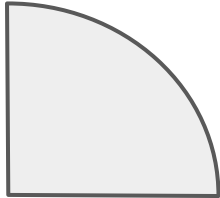


1. Introduction: Global parameterization

More complicated domains

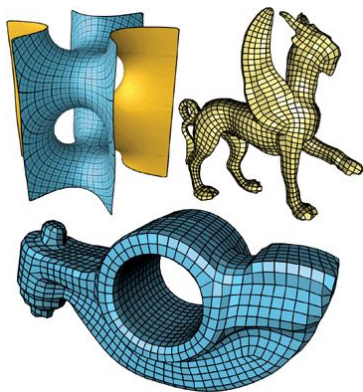
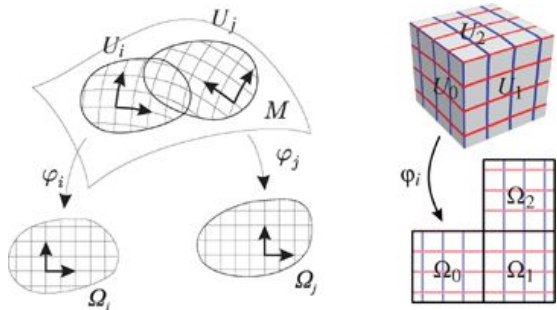


More complicated domains

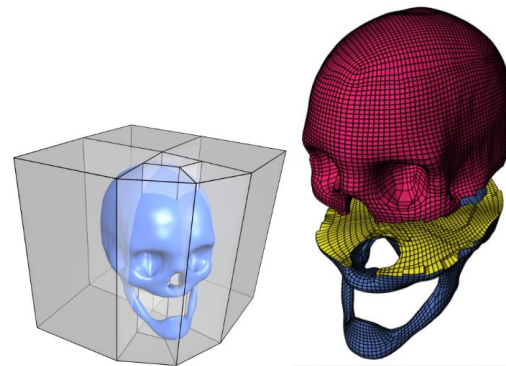
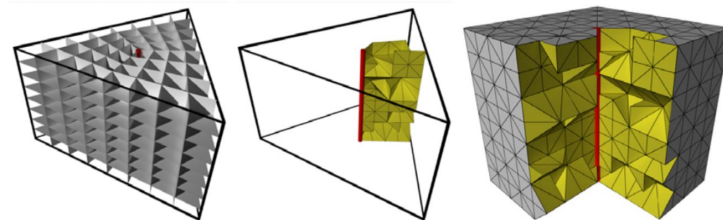


1. Introduction: Global parameterization

First implementations



[PGP, quadcover, mixed-Integer quadrangulation]

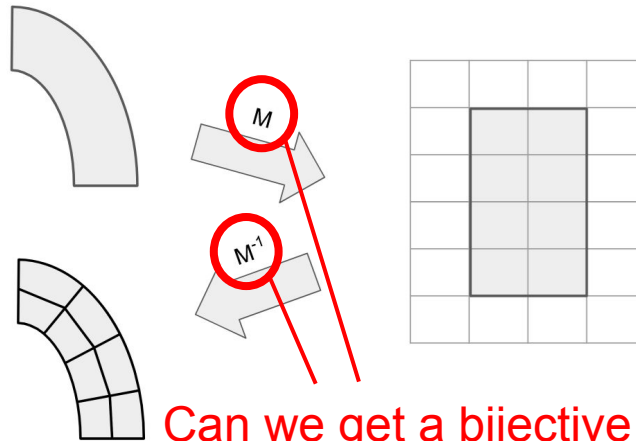


[Nieser et al, 2011]

Mapping bijectivity problems

2 reasons :

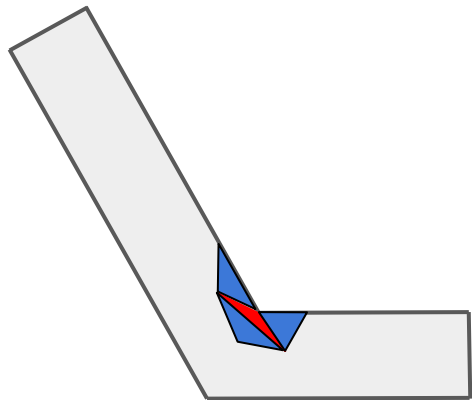
1. Interpolation of boundary constraints
2. Definition of boundary constraints



Can we get a bijective transformation ?

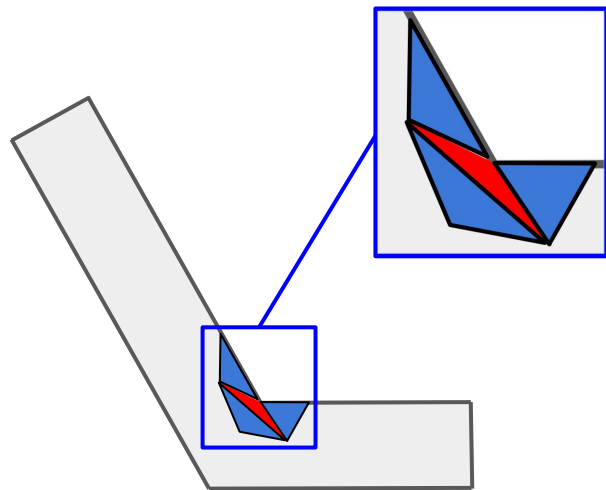
2.1 Bijectivity problems: 1st problem

Introducing 1st problem



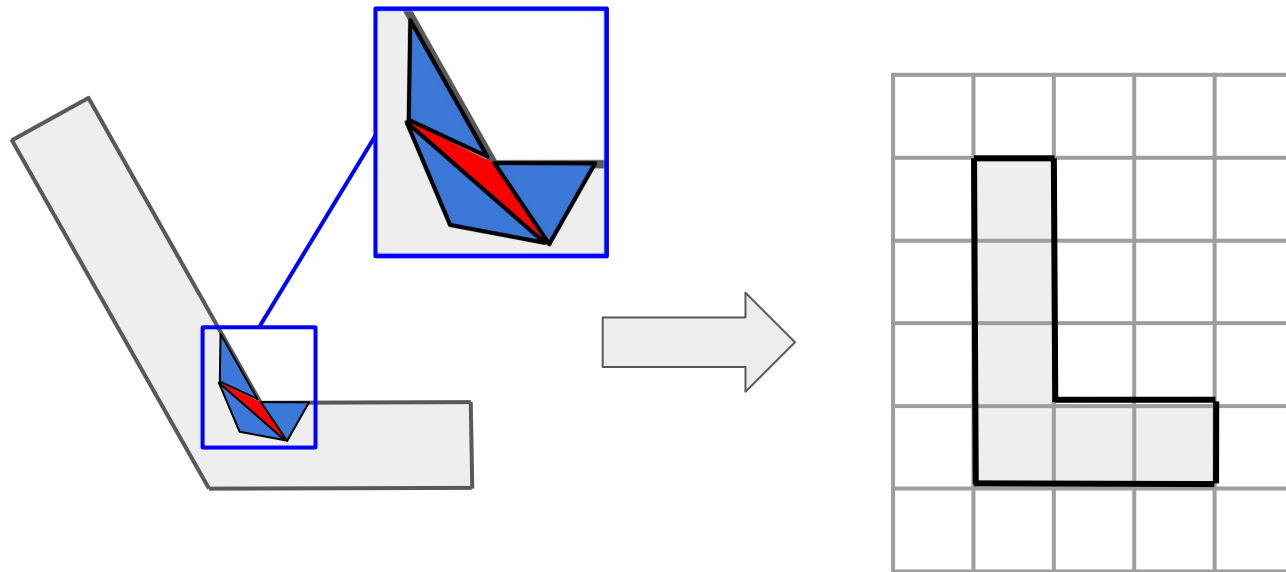
2.1 Bijectivity problems: 1st problem

Introducing 1st problem



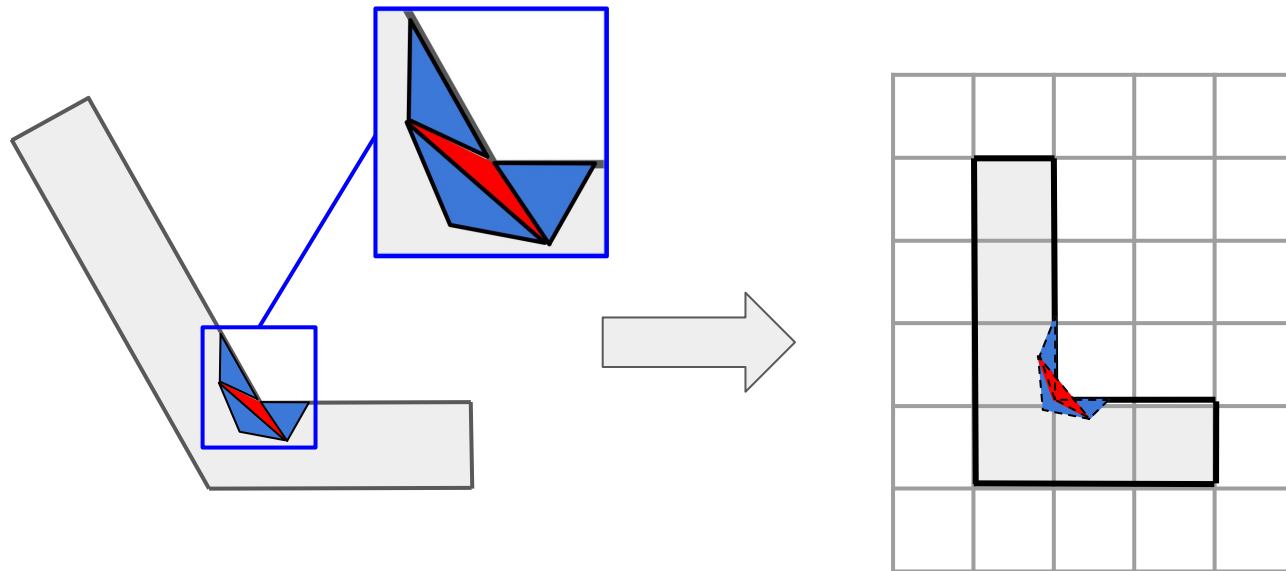
2.1 Bijectivity problems: 1st problem

Introducing 1st problem



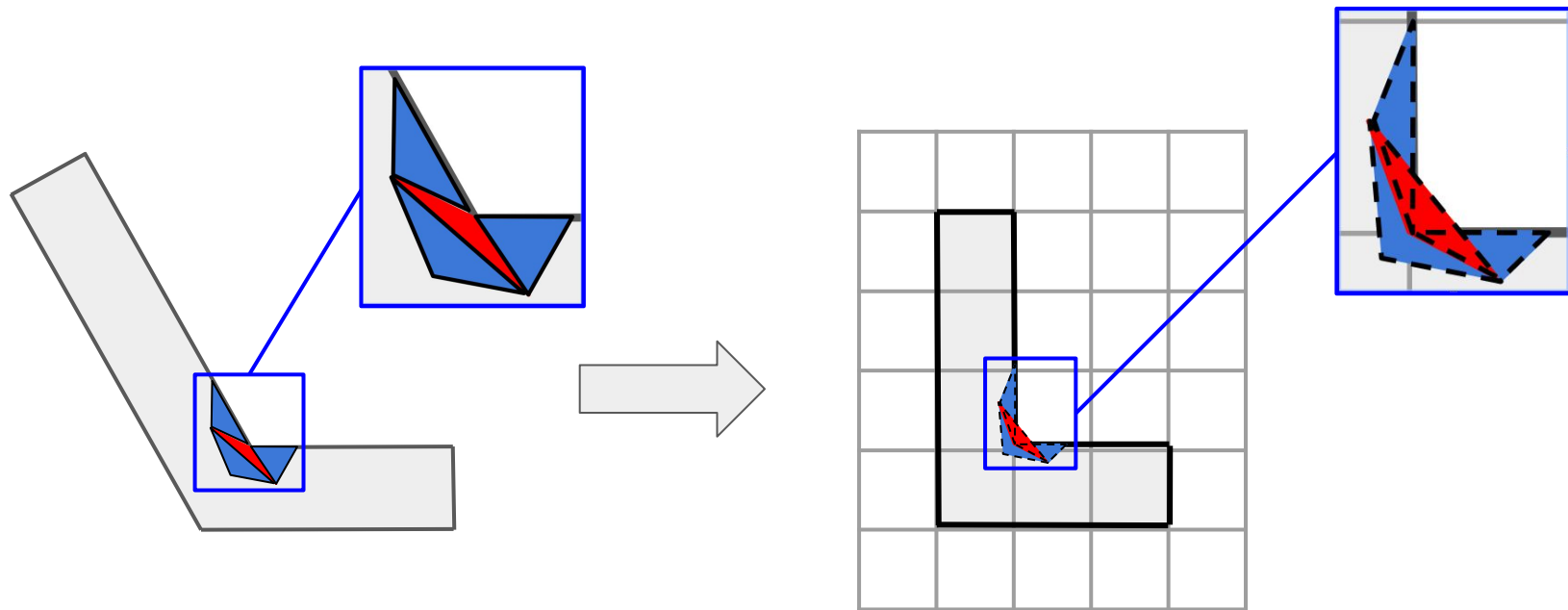
2.1 Bijectivity problems: 1st problem

Introducing 1st problem



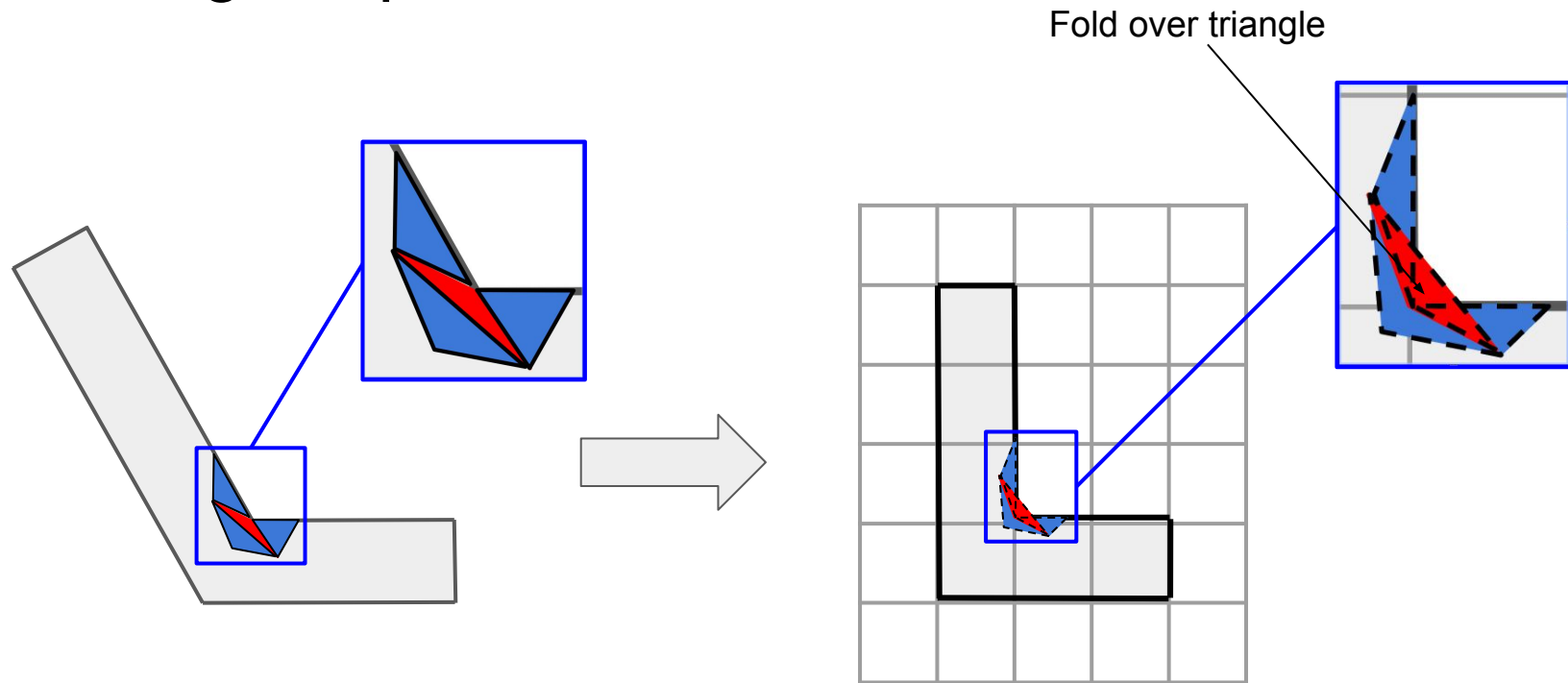
2.1 Bijectivity problems: 1st problem

Introducing 1st problem



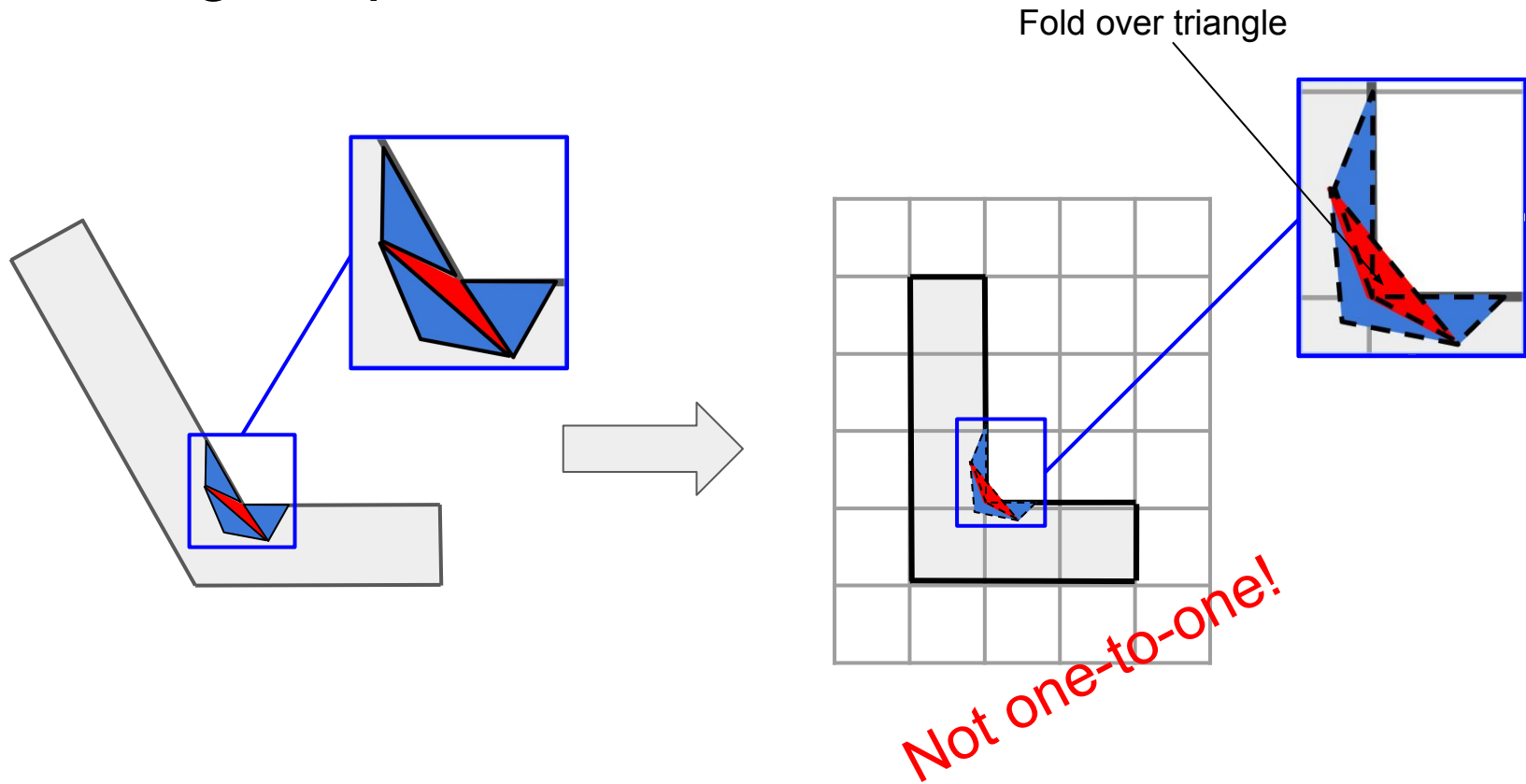
2.1 Bijectivity problems: 1st problem

Introducing 1st problem



2.1 Bijectivity problems: 1st problem

Introducing 1st problem



2.1 Bijectivity problems: 1st problem

Reality example



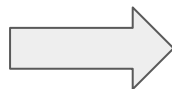
2.1 Bijectivity problems: 1st problem

Reality example



2.1 Bijectivity problems: 1st problem

Reality example

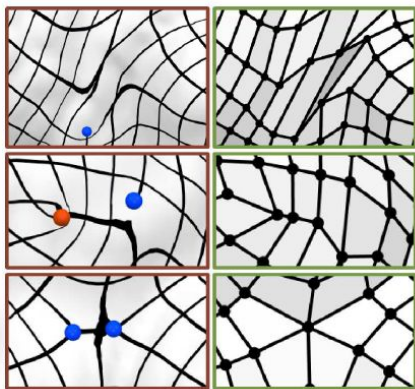
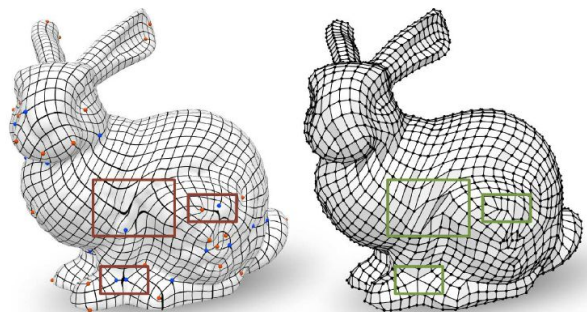


2.1 Bijectivity problems: 1st problem

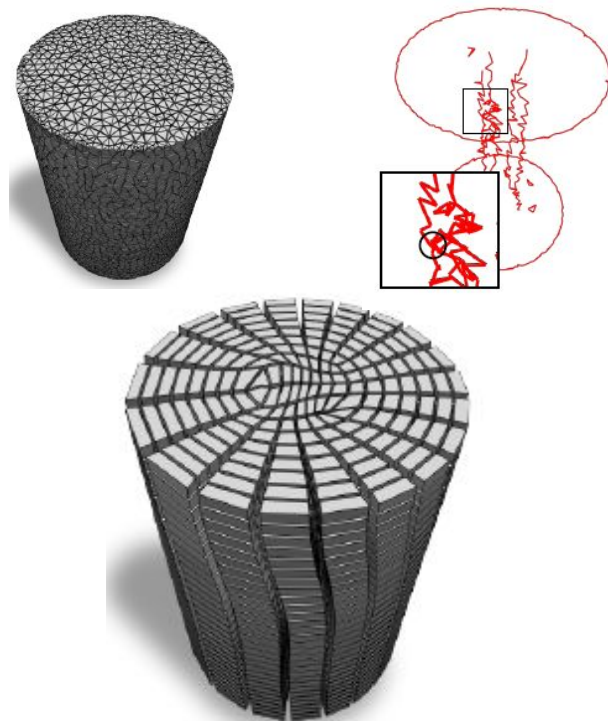
Solution

Allowing local non-bijectivity

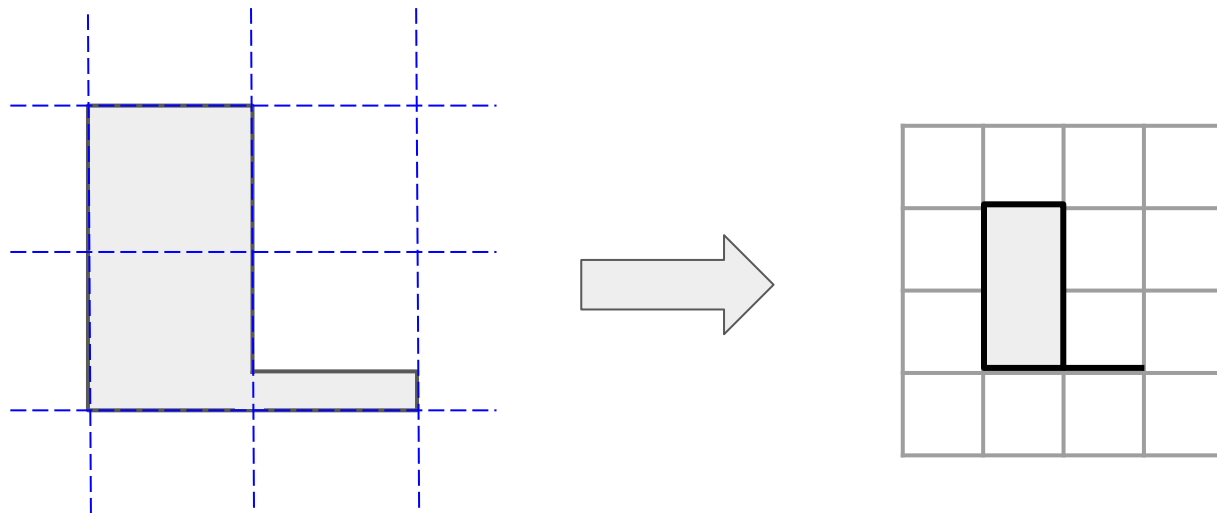
[Ebke et al, 2013]



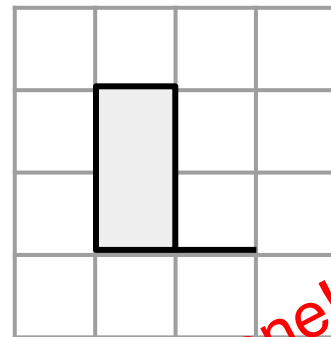
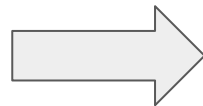
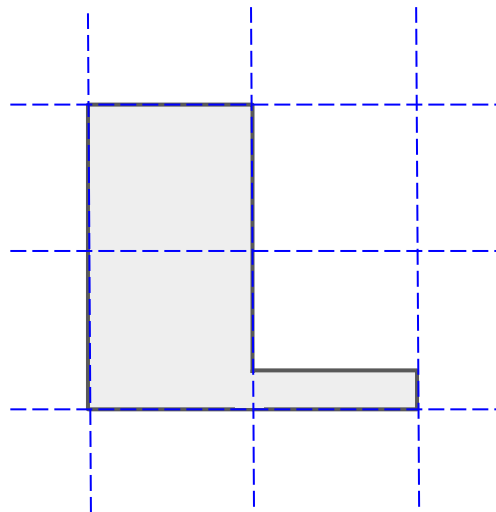
[Lyon et al, 2016]



Introducing 2nd problem



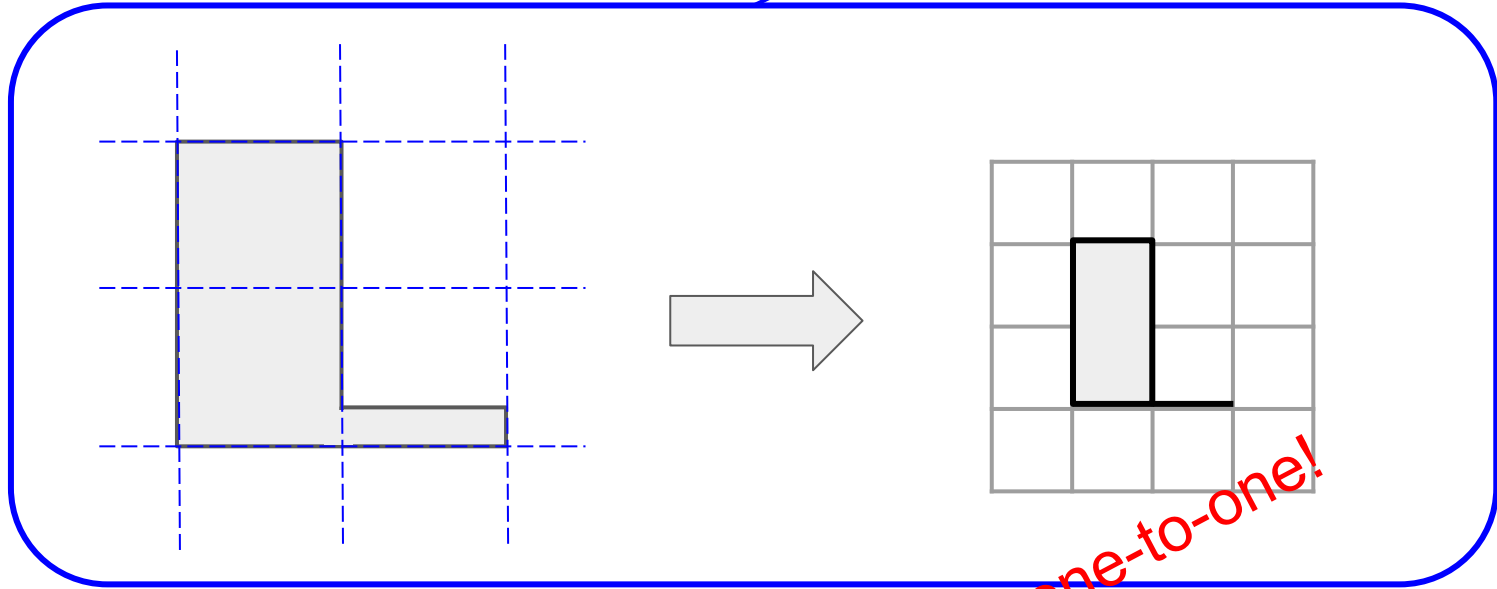
Introducing 2nd problem



Not one-to-one!

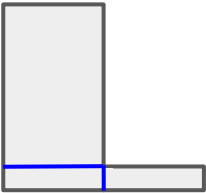
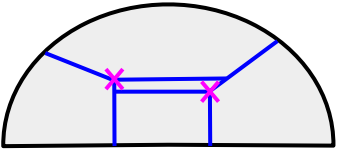
Introducing 2nd problem

What we will focus on



Not one-to-one!

Solutions

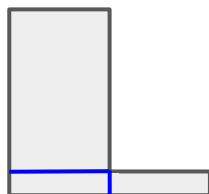
	Avoiding collapse on singularity-less models	General parameterization
2D		
3D		

Solutions

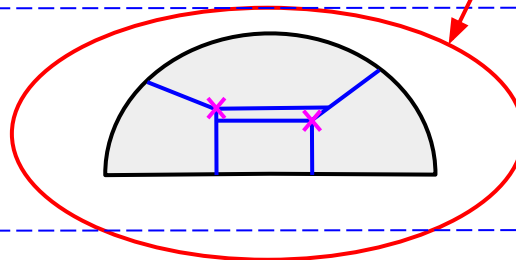
[Myles et al, 2012;2013;2014]
[Campen et al, 2015]

Avoiding collapse on singularity-less
models

2D

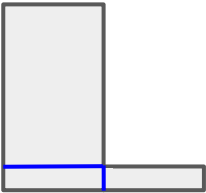
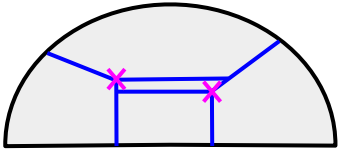



General parameterization

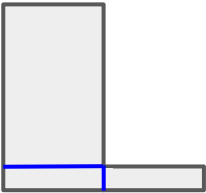
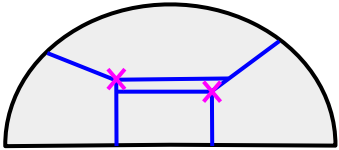


3D

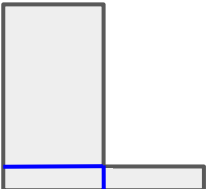
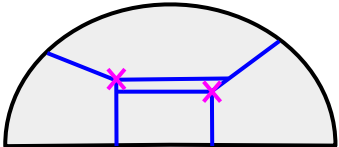
Solutions

	Avoiding collapse on singularity-less models	General parameterization
2D		 [Campen et al, 2015]
3D		

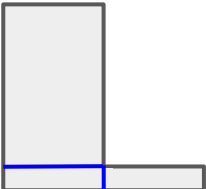
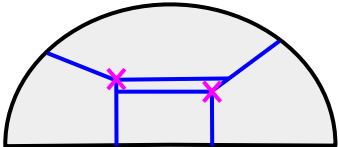
Solutions

	Avoiding collapse on singularity-less models	General parameterization
2D		 [Campen et al, 2015]
↓ ?		↓ ?
3D		[Kowalski et al, 2014;2016] ?

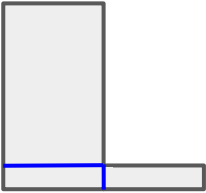
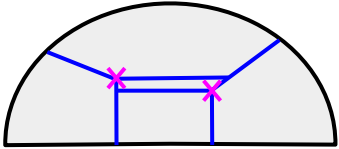
Solutions

	Avoiding collapse on singularity-less models	General parameterization
2D		 [Campen et al, 2015]
3D		No good result yet

Solutions

	Avoiding collapse on singularity-less models	General parameterization
2D		 [Campen et al, 2015]
3D	What we propose here	No good result yet

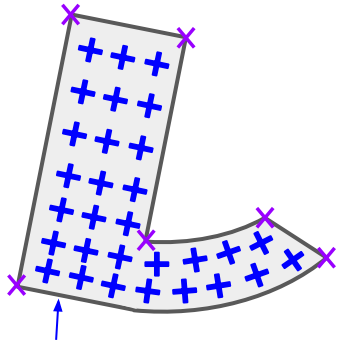
Solutions

	Avoiding collapse on singularity-less models	General parameterization
2D		 [Campen et al, 2015]
3D	What we propose here	No good result yet

Further work

2.2 Bijectivity problems: 2nd problem

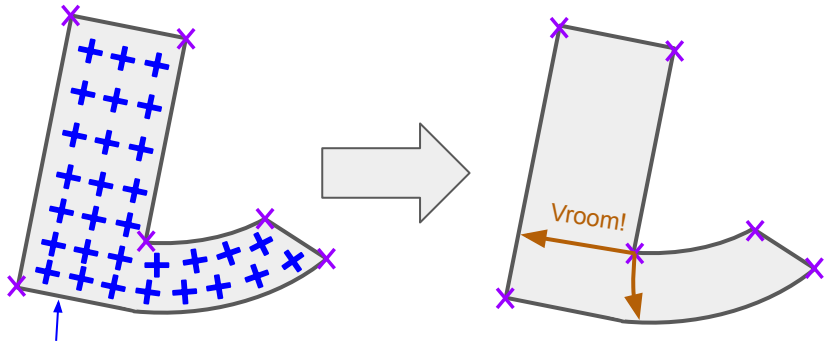
2d solution: motorcycles



Orientation that we
want for our hexes

2.2 Bijectivity problems: 2nd problem

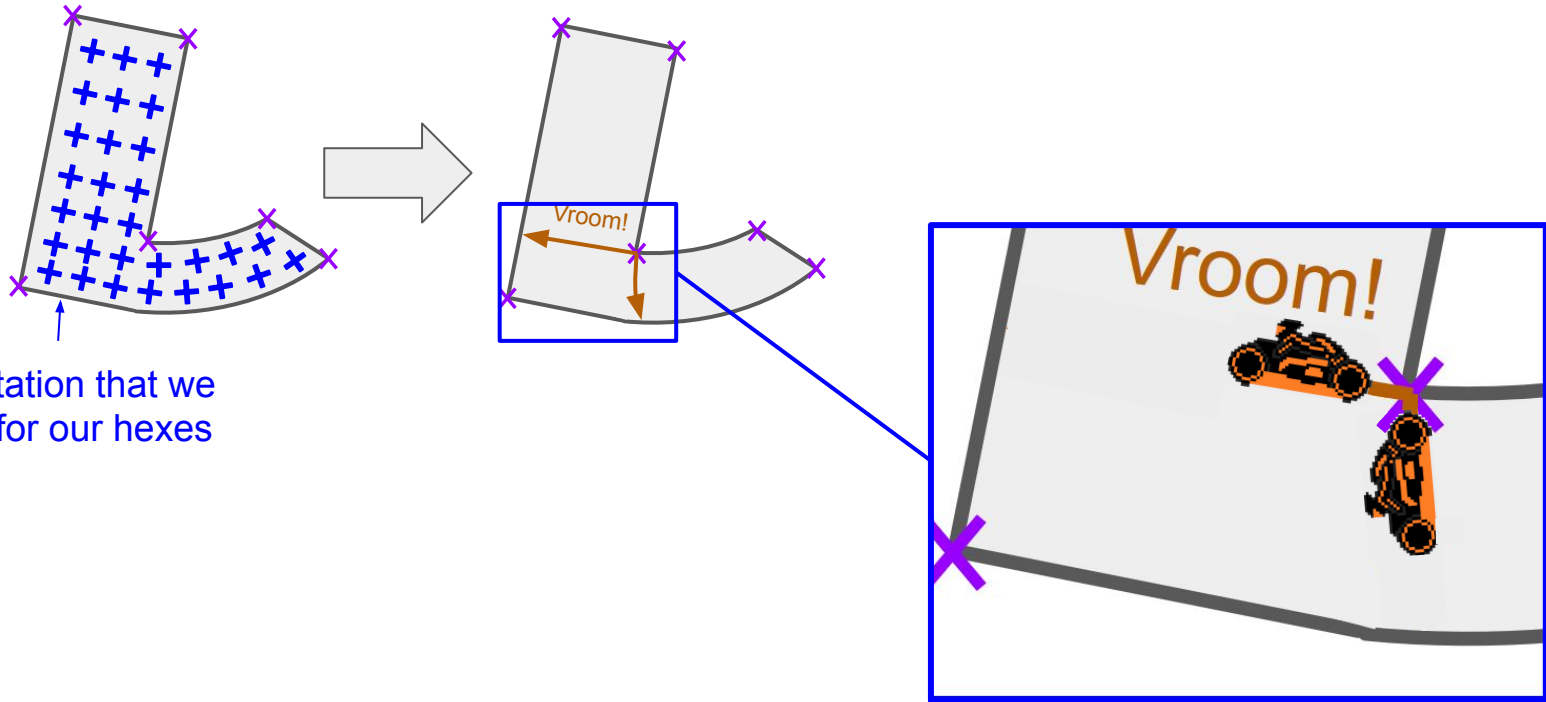
2d solution: motorcycles



Orientation that we
want for our hexes

2.2 Bijectivity problems: 2nd problem

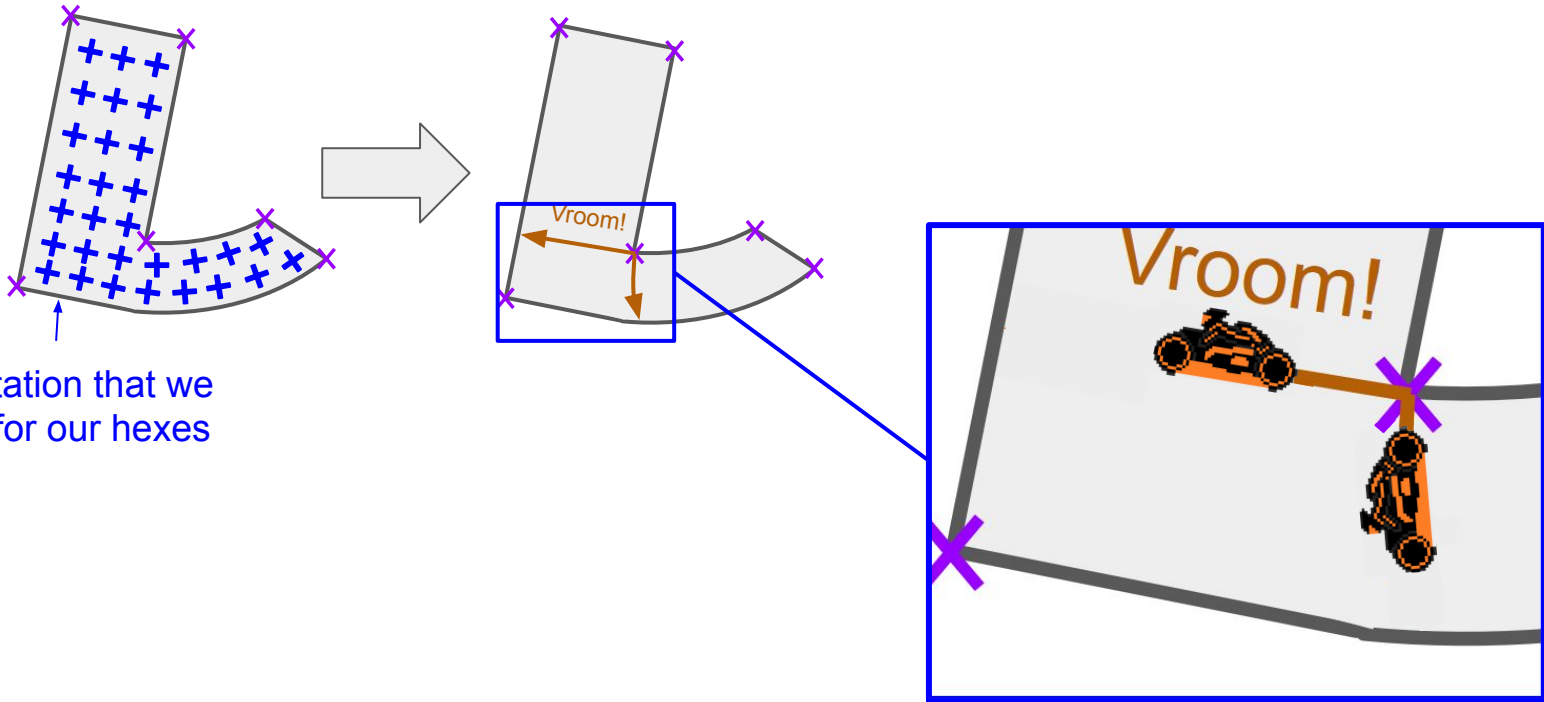
2d solution: motorcycles



Orientation that we
want for our hexes

2.2 Bijectivity problems: 2nd problem

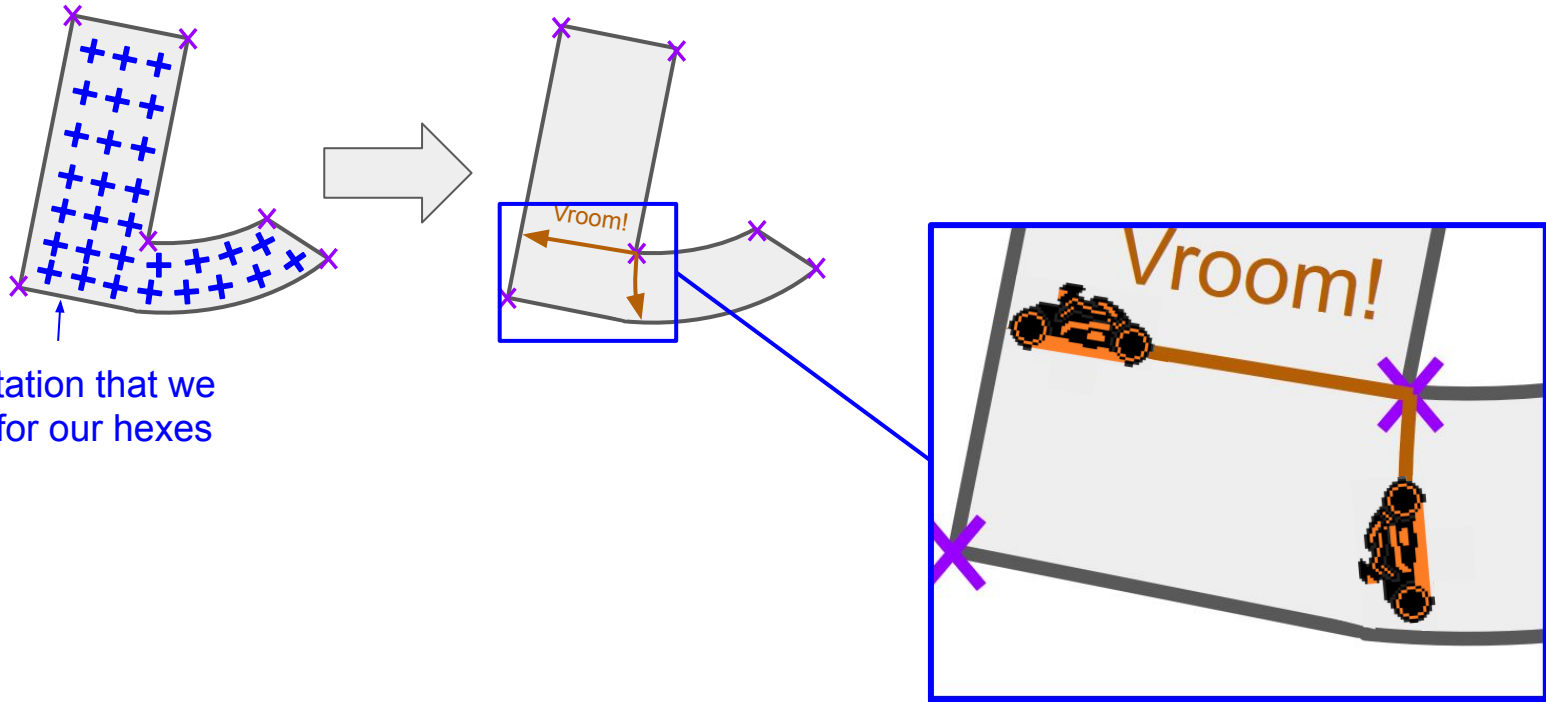
2d solution: motorcycles



Orientation that we
want for our hexes

2.2 Bijectivity problems: 2nd problem

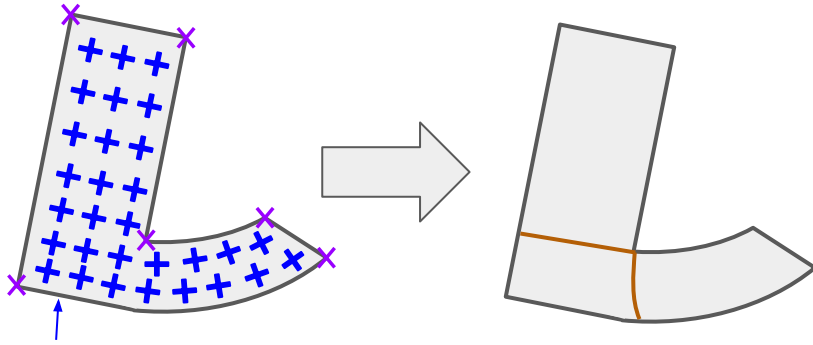
2d solution: motorcycles



Orientation that we want for our hexes

2.2 Bijectivity problems: 2nd problem

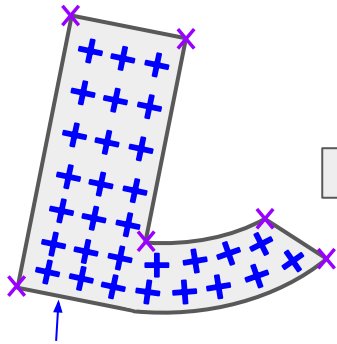
2d solution: motorcycles



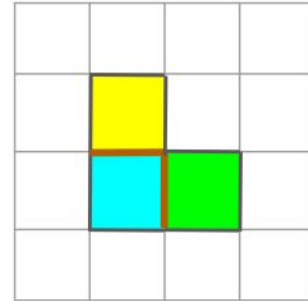
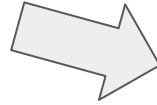
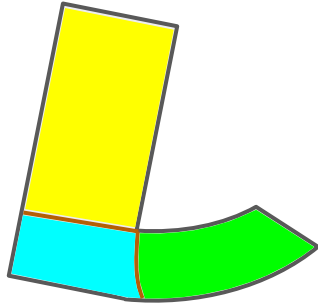
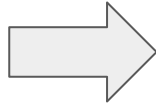
Orientation that we
want for our hexes

2.2 Bijectivity problems: 2nd problem

2d solution: motorcycles

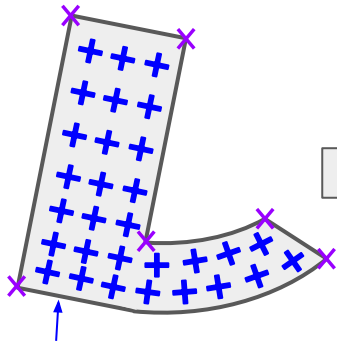


Orientation that we want for our hexes

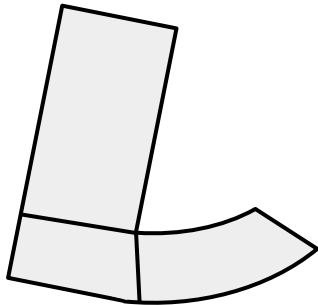
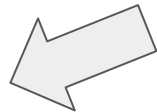
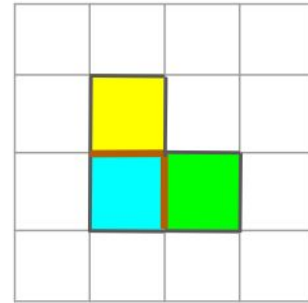
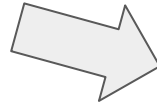
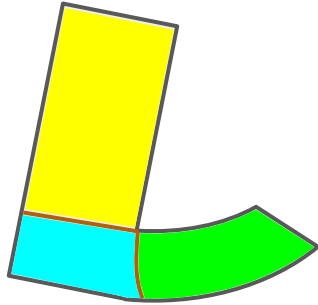
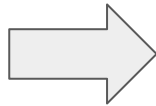


2.2 Bijectivity problems: 2nd problem

2d solution: motorcycles

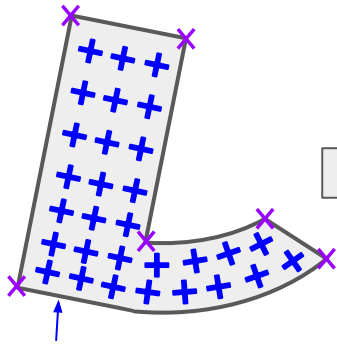


Orientation that we want for our hexes

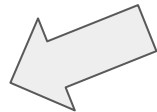
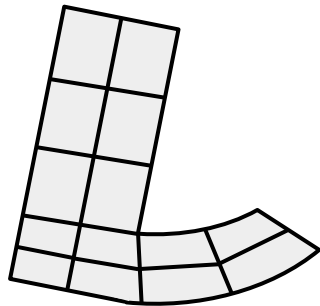
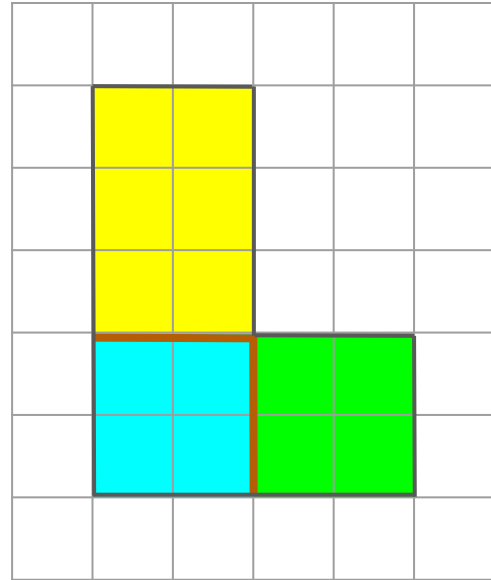
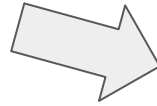
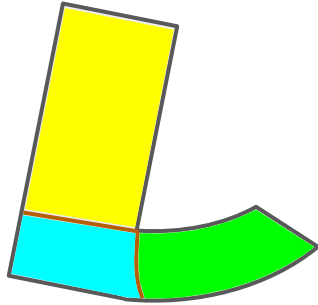
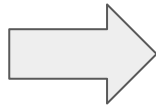


2.2 Bijectivity problems: 2nd problem

2d solution: motorcycles

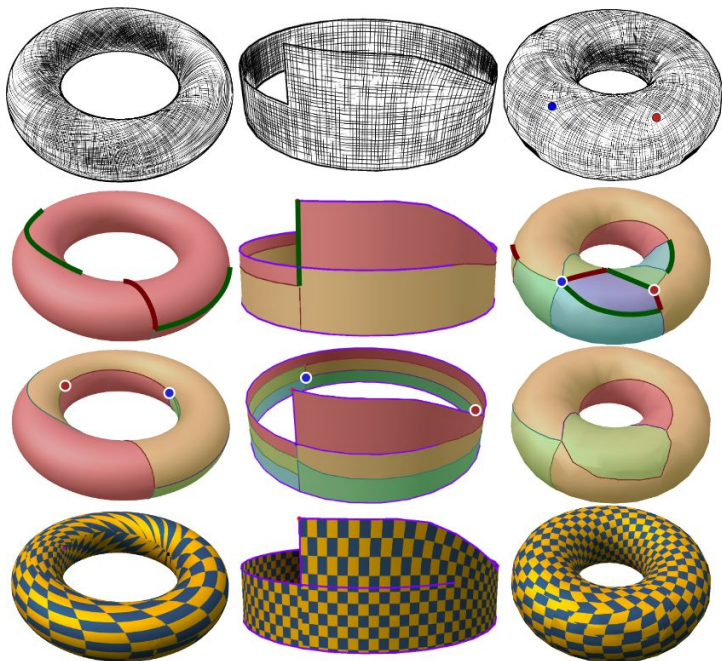


Orientation that we want for our hexes

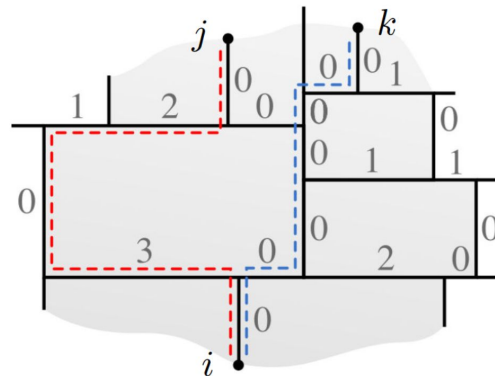
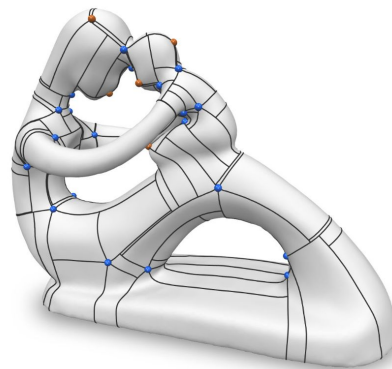


2.2 Bijectivity problems: 2nd problem

2d solution: T-junctions and no boundary

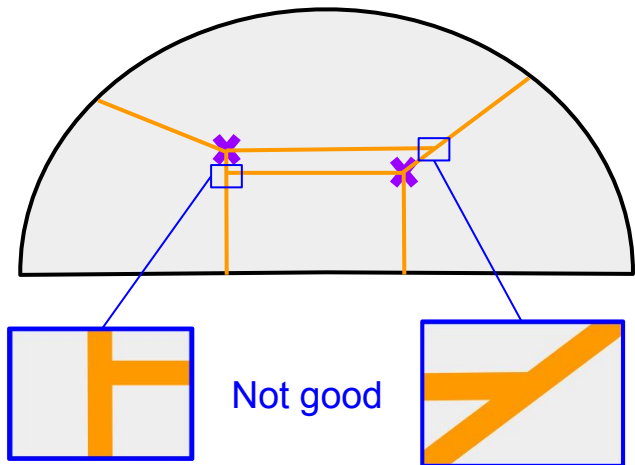


[Myles et al, 2012;2013;2014]

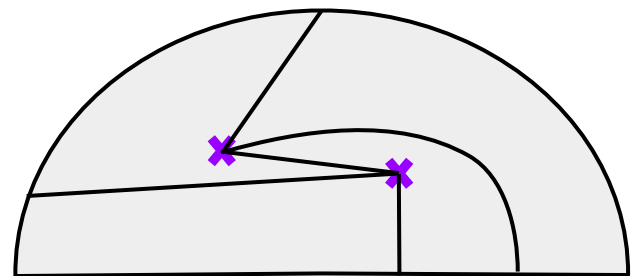
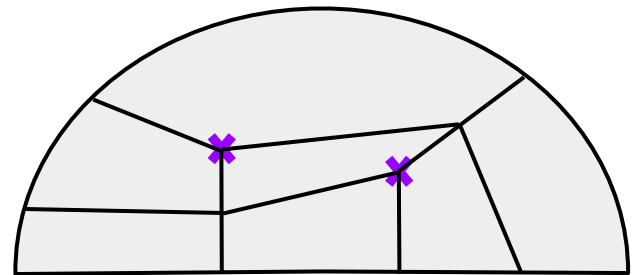
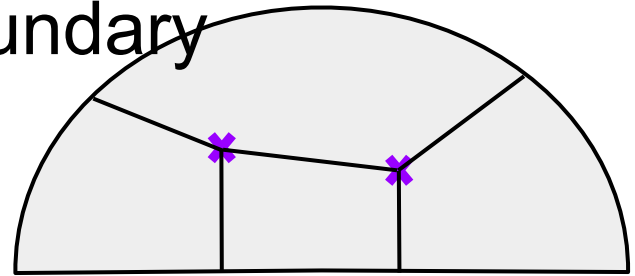
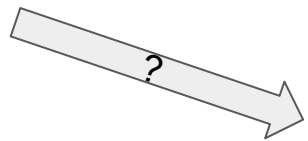
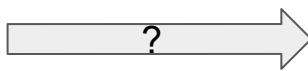
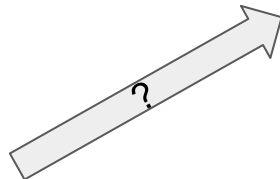


[Campen et al, 2015]

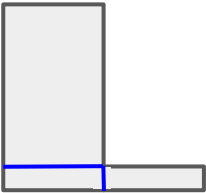
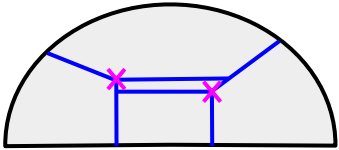
2d solution: T-junctions and no boundary



Combinatorial optimisation

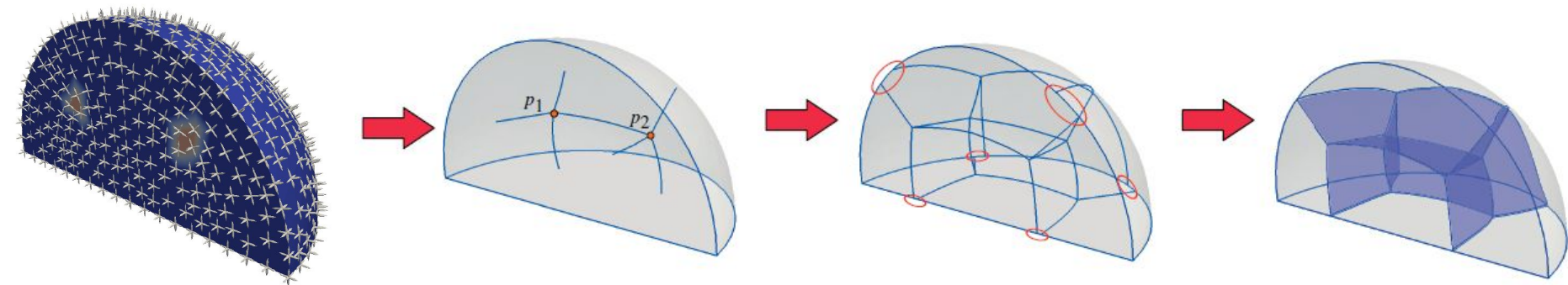


3d solution: ?

	Avoiding collapse on singularity-less models	General parameterization
2D		 [Campen et al, 2015]
3D	What we propose here	No good result yet

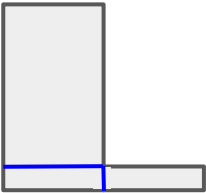
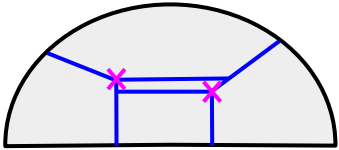
2.2 Bijectivity problems: 2nd problem

3d solution: ?



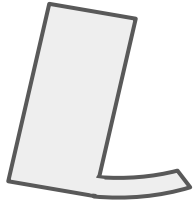
[Kowalski et al, 2014;2016]

3d solution: solving a sub-problem

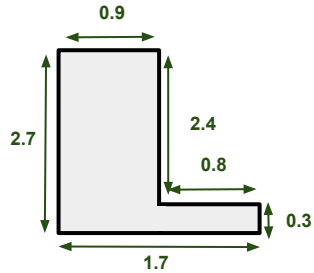
	Avoiding collapse on singularity-free models	General parameterization
2D		 [Campen et al, 2015]
3D	What we propose here	No good result yet

3.1 Studying a sub-problem: Introducing

Naive polycube approach

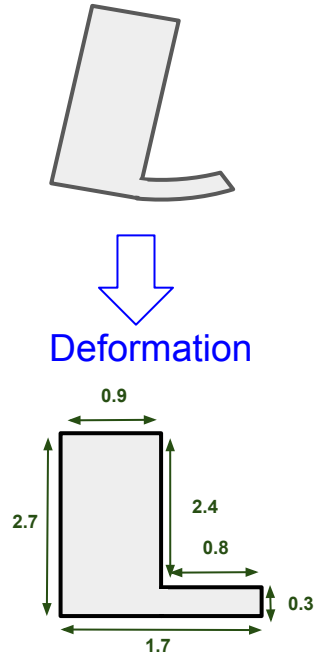


Deformation



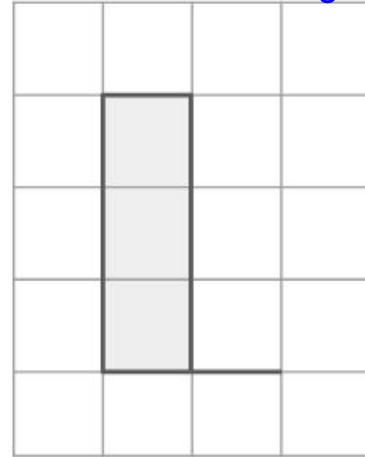
3.1 Studying a sub-problem: Introducing

Naive polycube approach

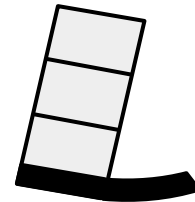


Snapping

Deformation on a grid

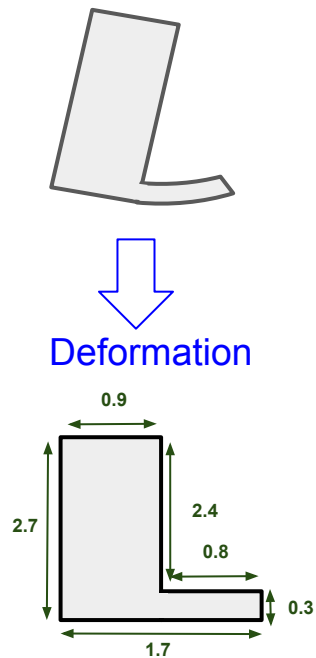


Qex - Hexex



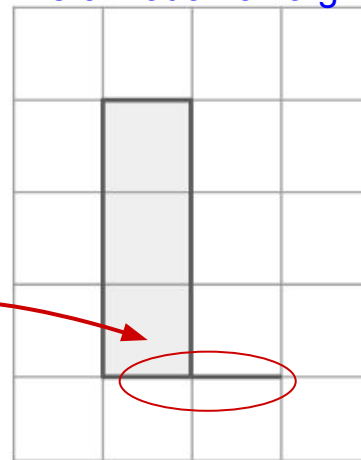
3.1 Studying a sub-problem: Introducing

Naive polycube approach

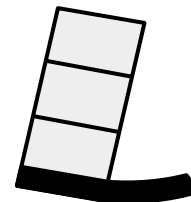


Snapping

Deformation on a grid

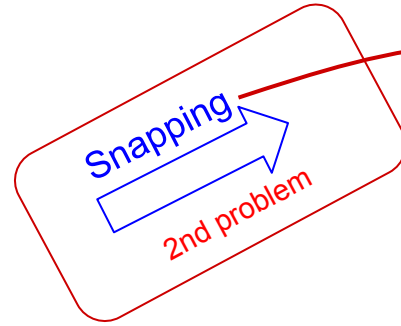
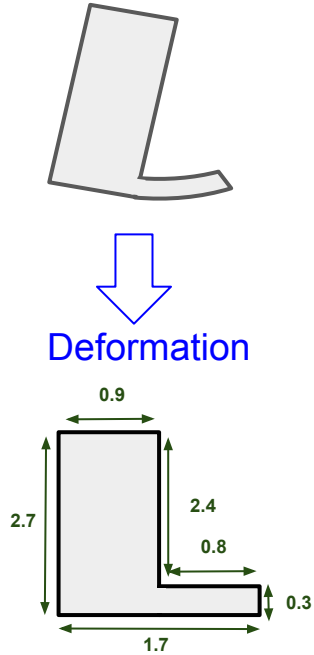


Qex - Hexex

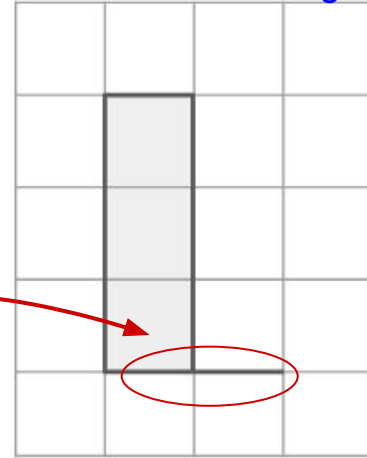


3.1 Studying a sub-problem: Introducing

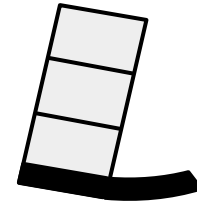
Naive polycube approach



Deformation on a grid

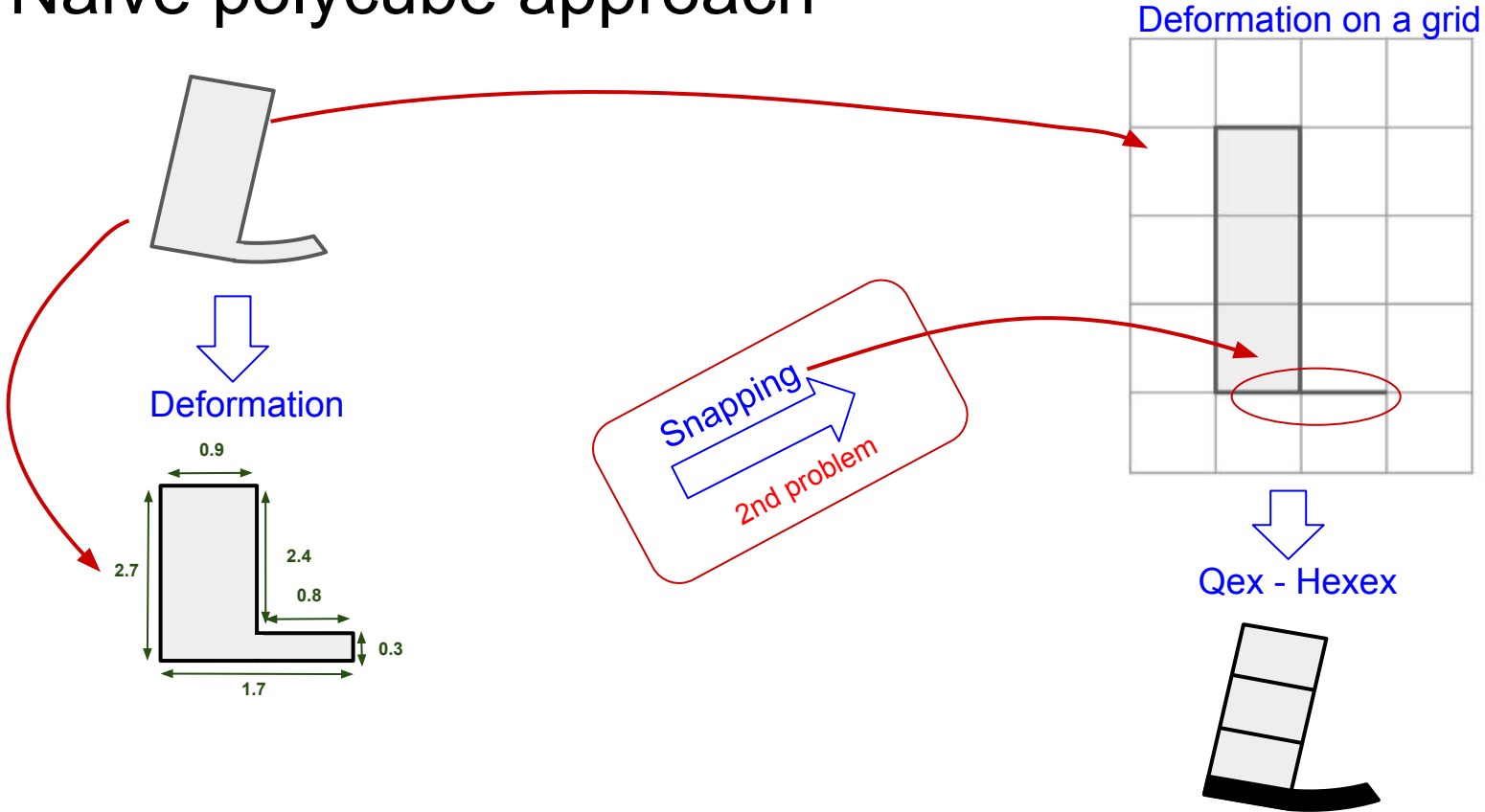


Qex - Hexex



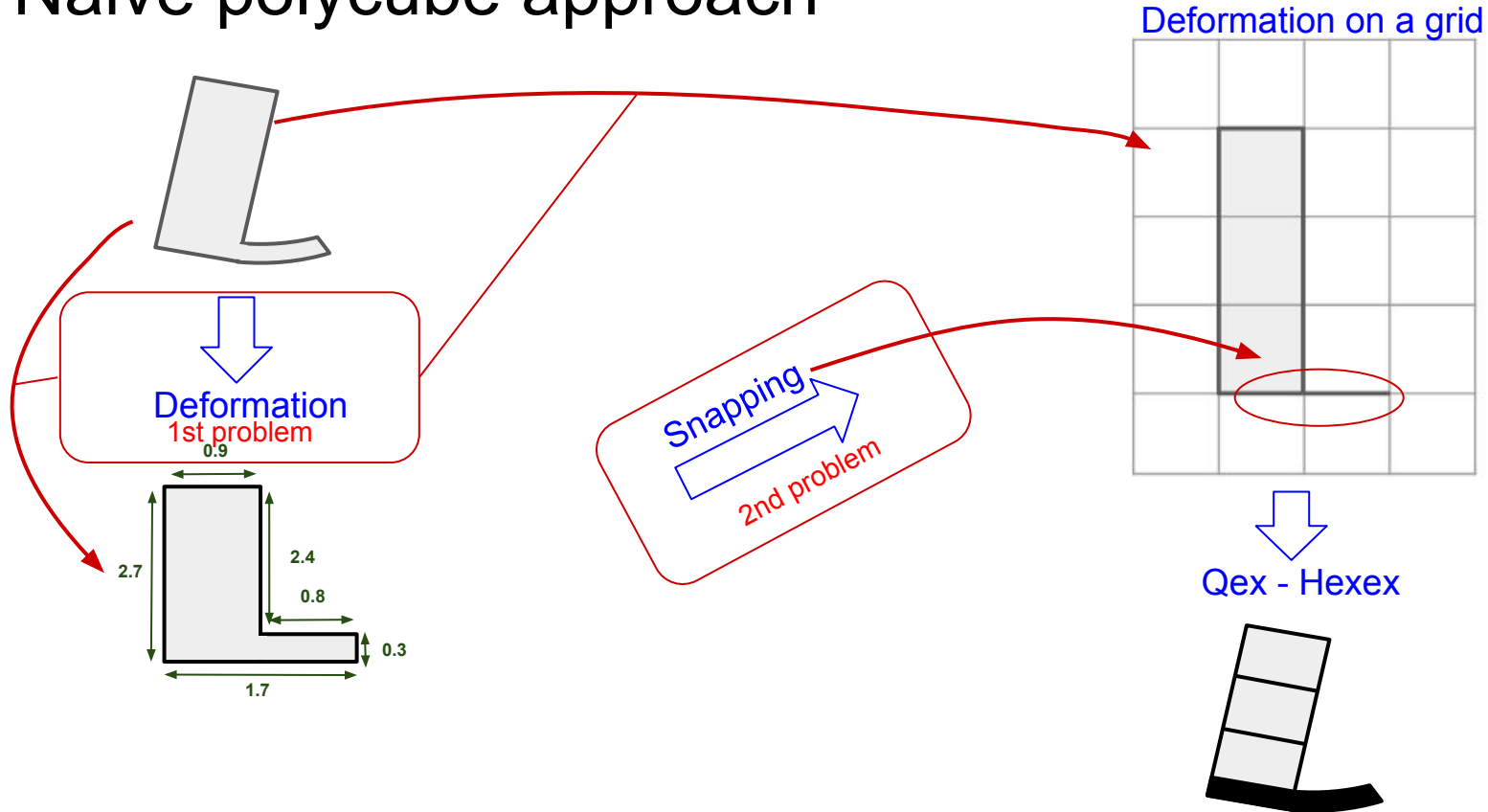
3.1 Studying a sub-problem: Introducing

Naive polycube approach



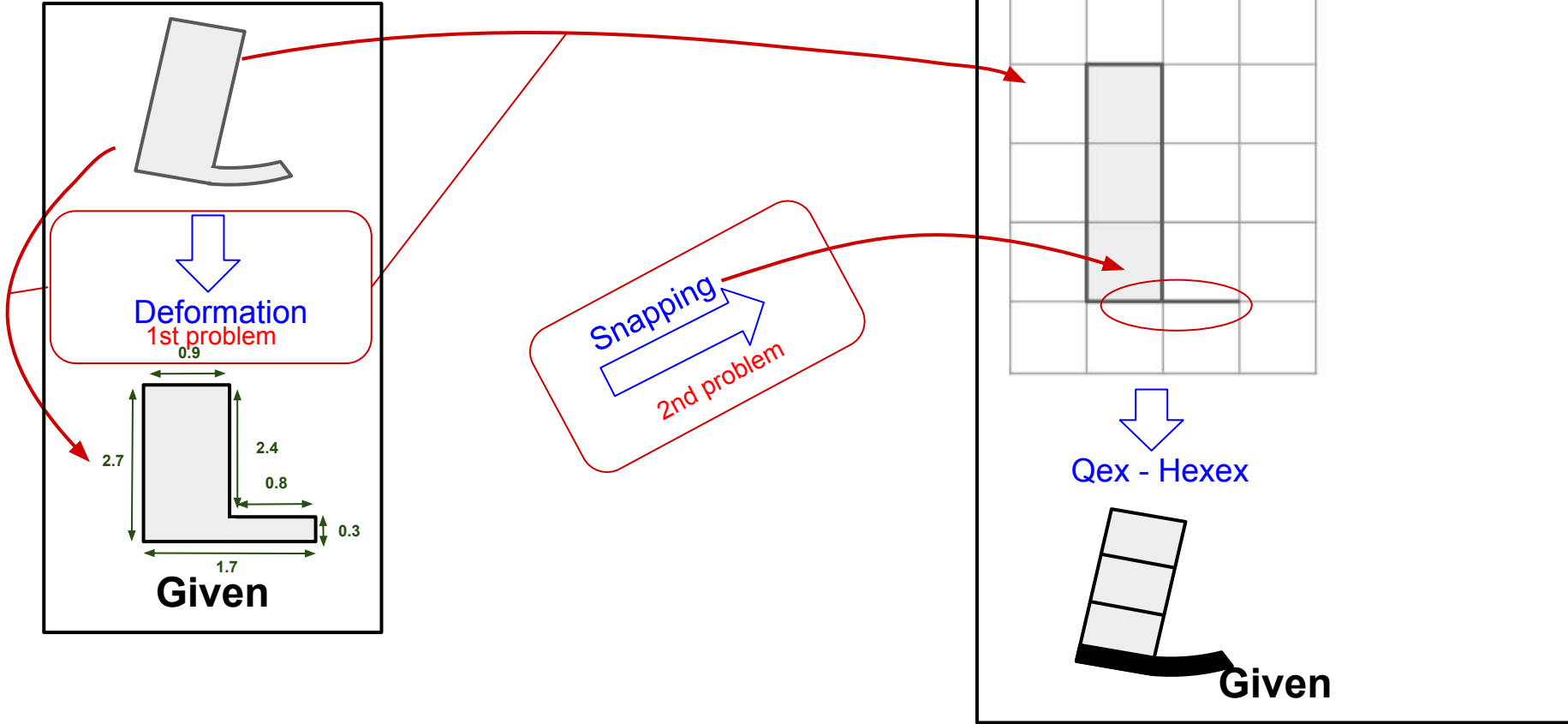
3.1 Studying a sub-problem: Introducing

Naive polycube approach



3.1 Studying a sub-problem: Introducing

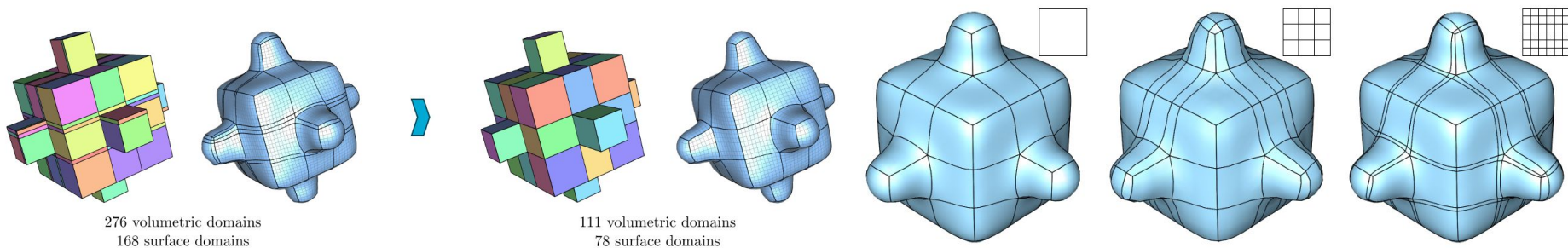
Naive polycube approach



3.1 Studying a sub-problem: Solving

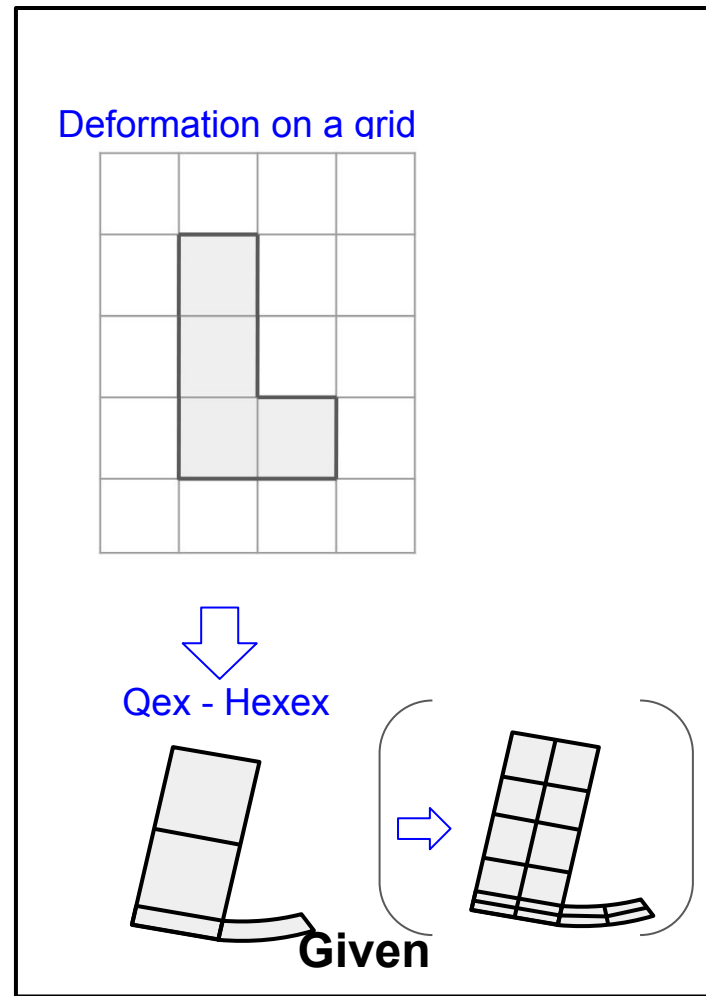
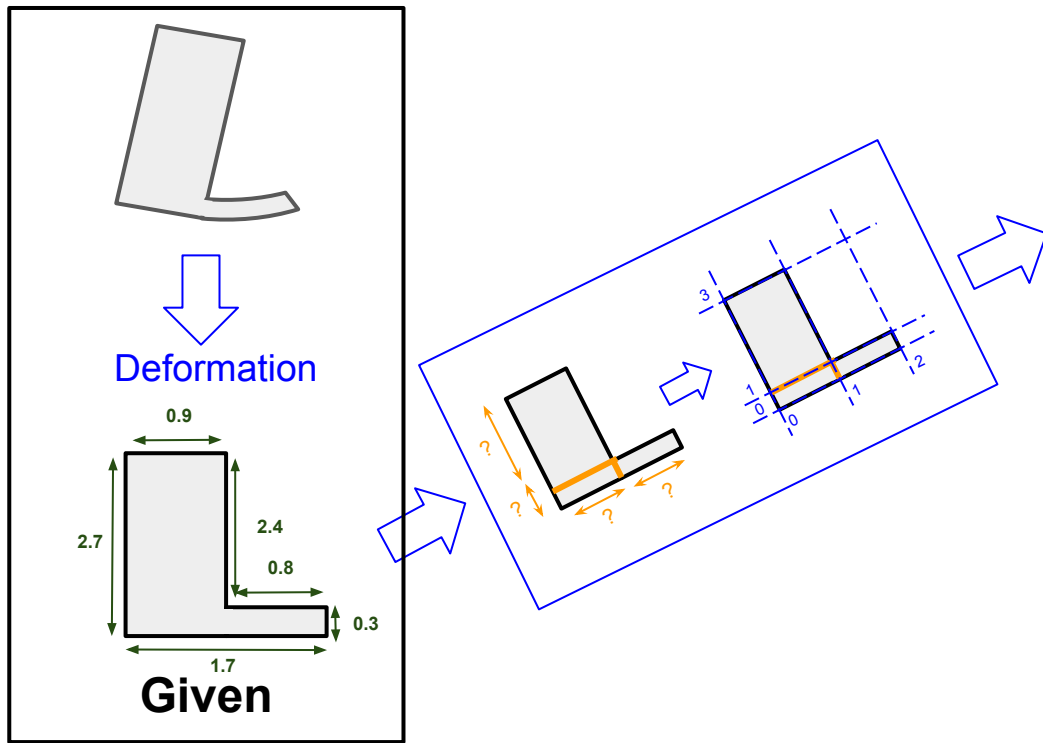
Related work

- **[Kowalski et al, 2014;2016]**
- **[Cherchi et al., 2016]** : Improve a given polycube by focusing on corner pairing to improve edge alignment :



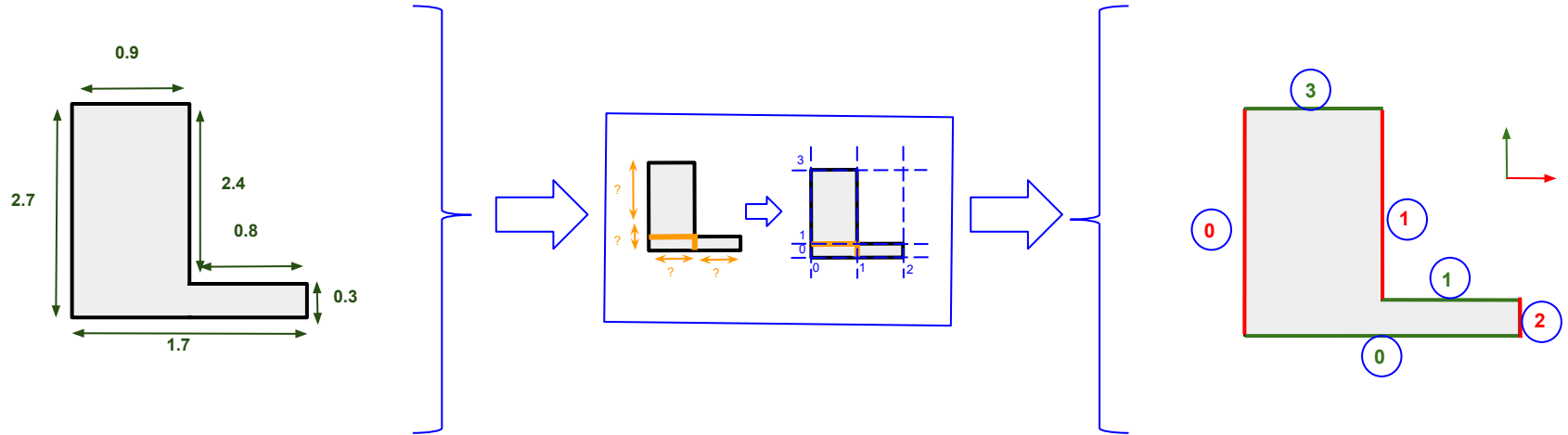
- **[Chen et al, 2019]** : constraint to no collapse edges of the polycubes

Our improvement



3.2 Studying a sub-problem: Our improvement

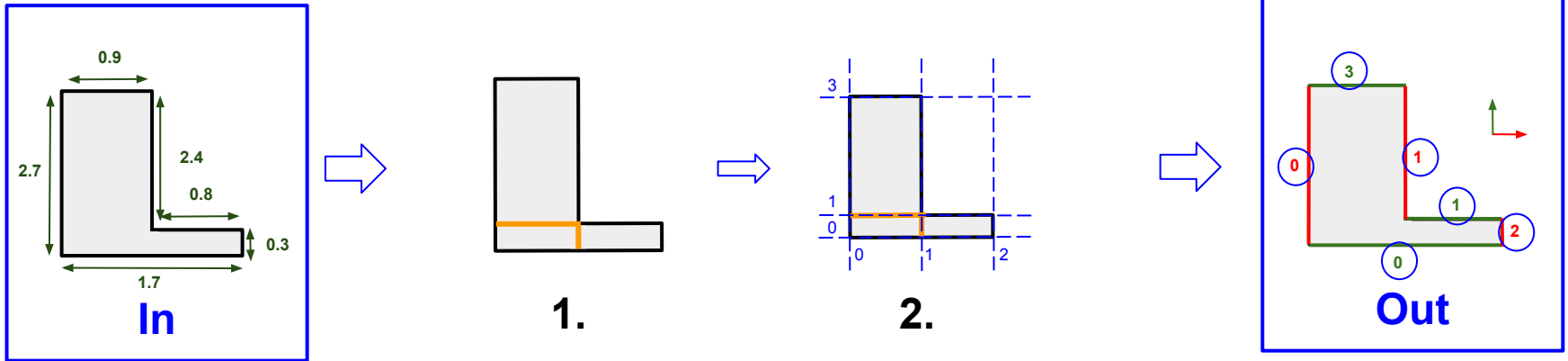
Our improvement



4. Our method: Introduction

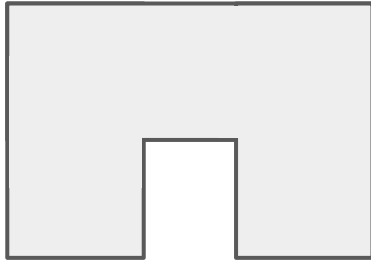
Our method

1. Generate combinatorial information with a motorcycle-like method
2. Finding the best set of integers respecting the combinatorics



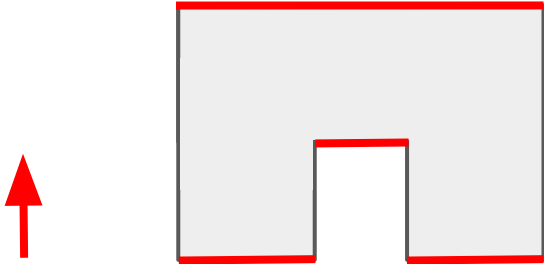
4.1 Our method: First polycube generation

Using motorcycles



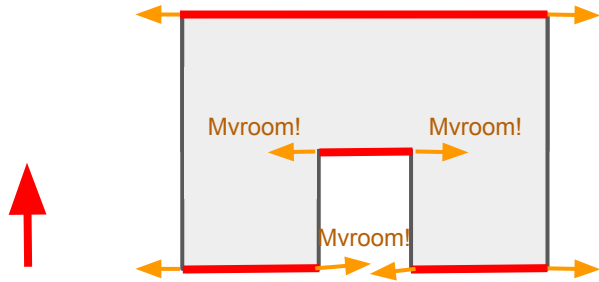
4.1 Our method: First polycube generation

Using motorcycles



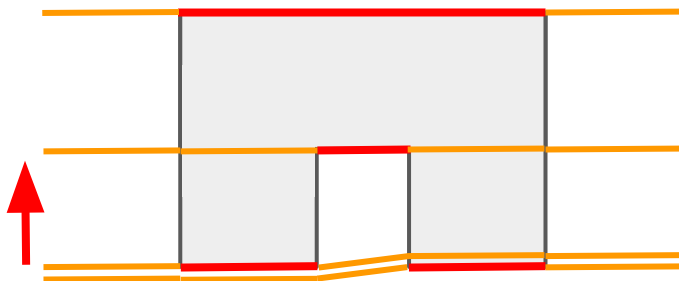
4.1 Our method: First polycube generation

Using motorcycles



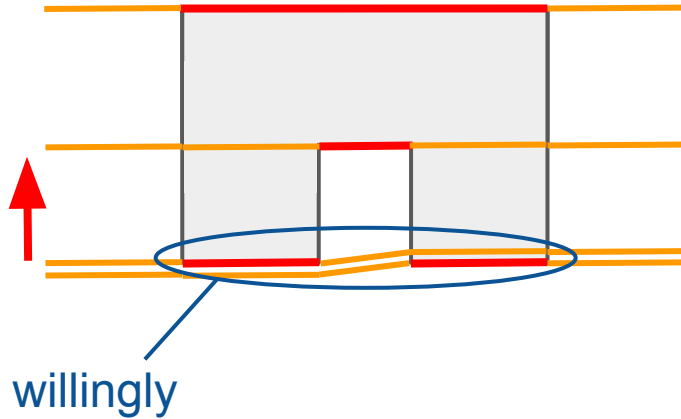
4.1 Our method: First polycube generation

Using motorcycles



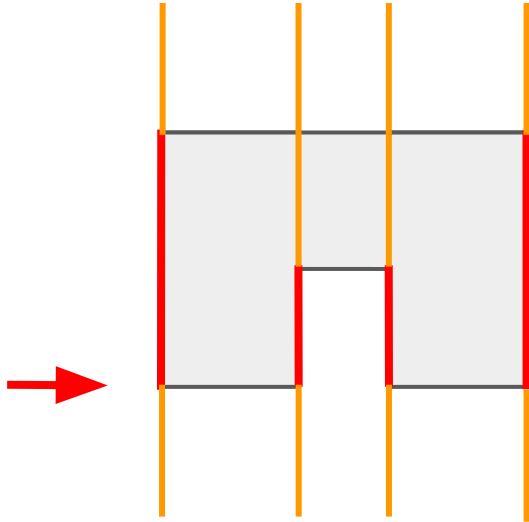
4.1 Our method: First polycube generation

Using motorcycles



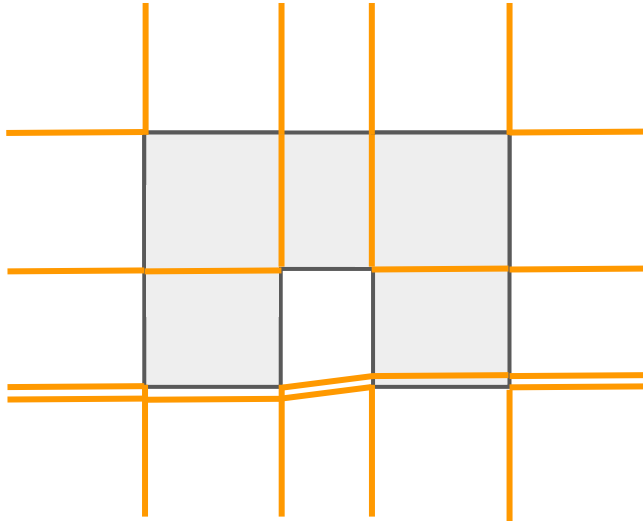
4.1 Our method: First polycube generation

Using motorcycles



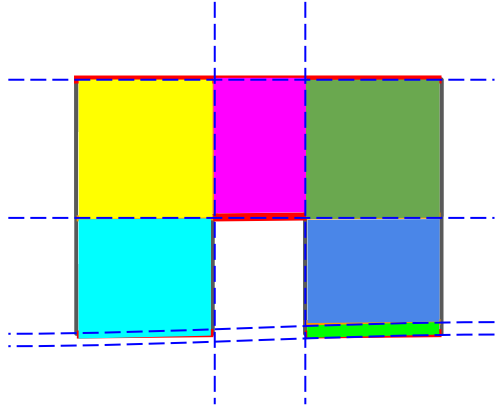
4.1 Our method: First polycube generation

Using motorcycles



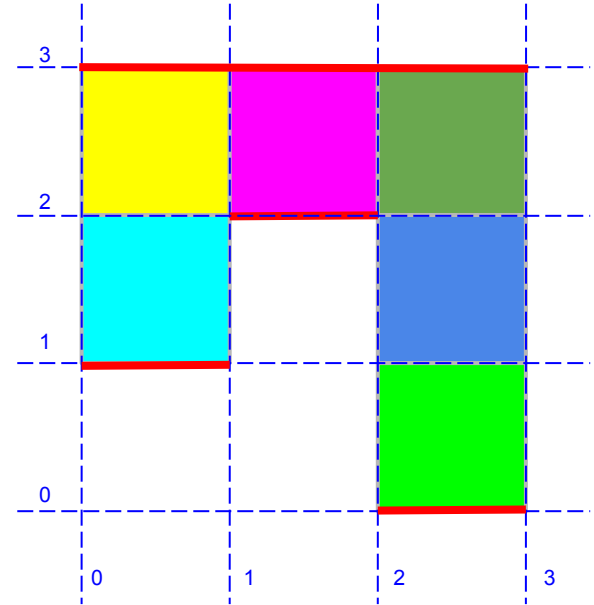
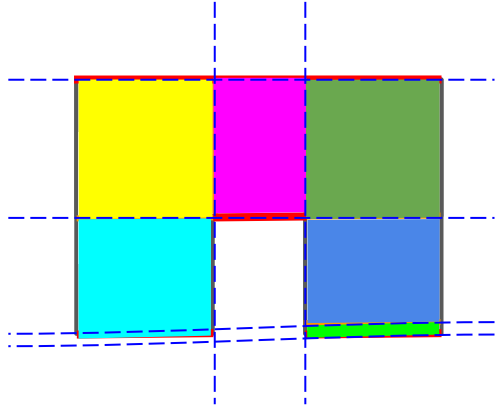
4.1 Our method: First polycube generation

Using motorcycles



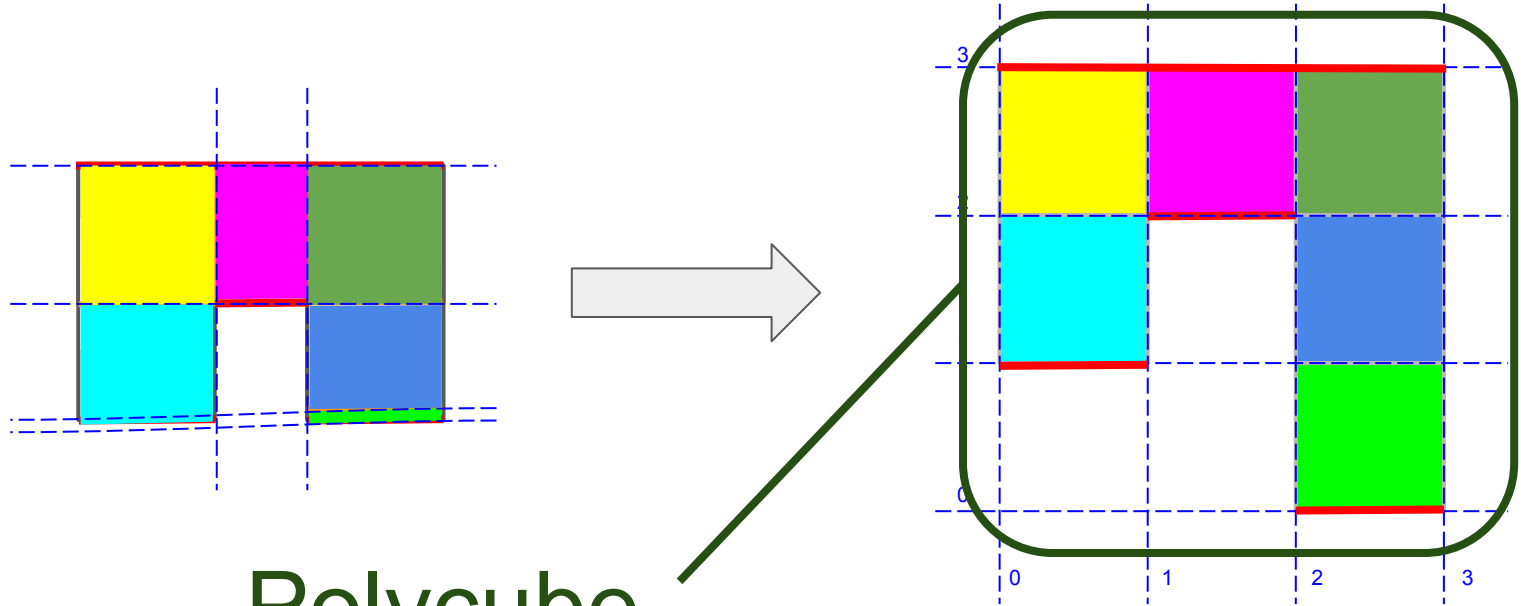
4.1 Our method: First polycube generation

Using motorcycles



4.1 Our method: First polycube generation

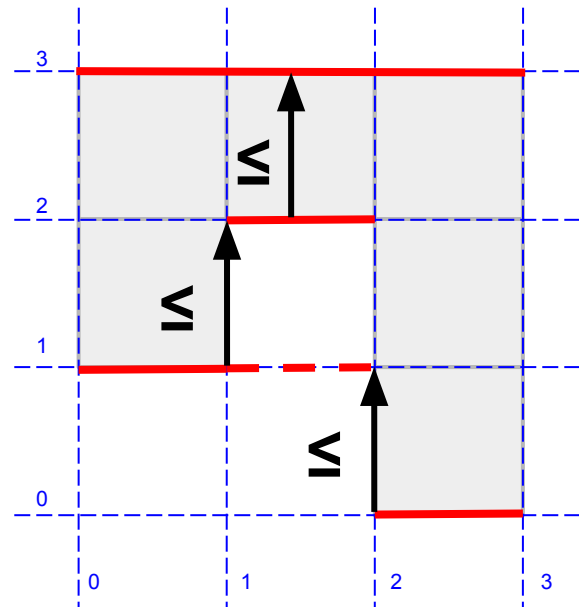
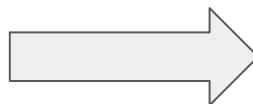
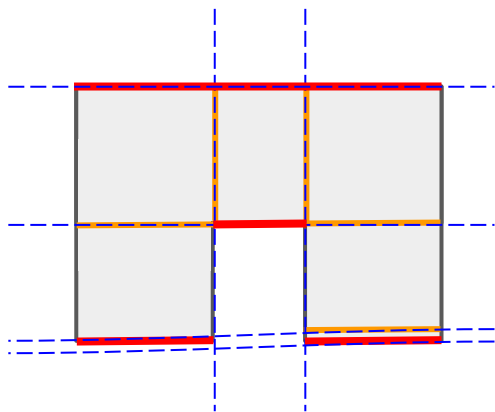
Using motorcycles



Polycube

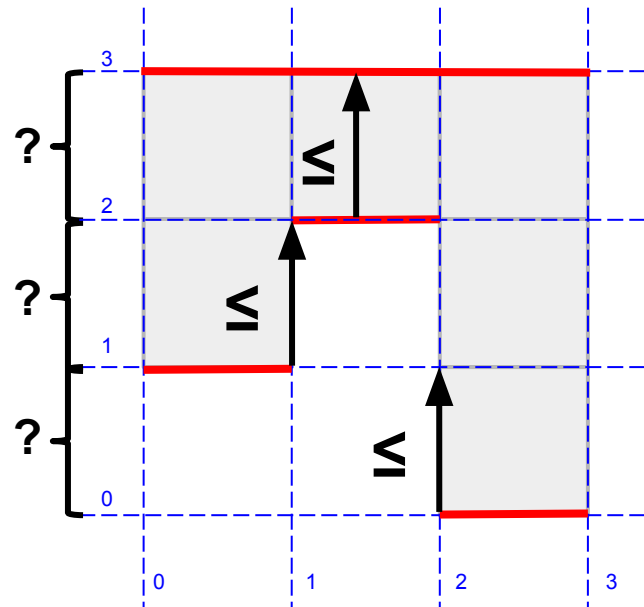
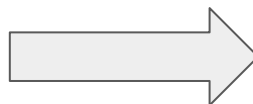
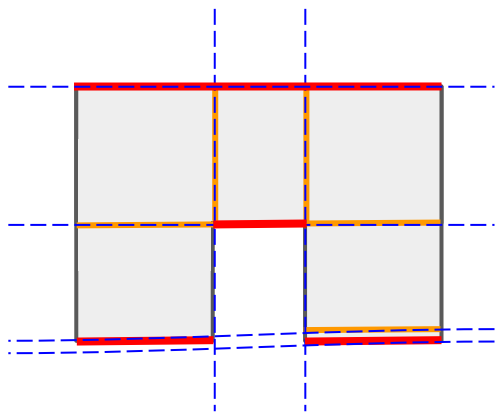
4.1 Our method: First polycube generation

Total order



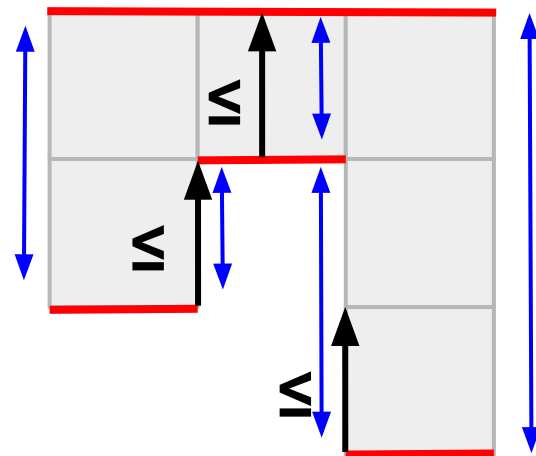
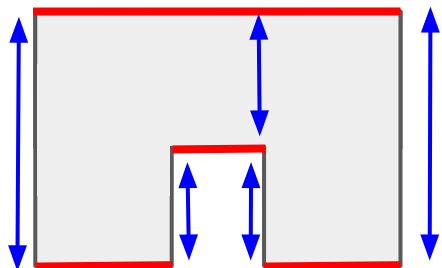
4.1 Our method: First polycube generation

Total order



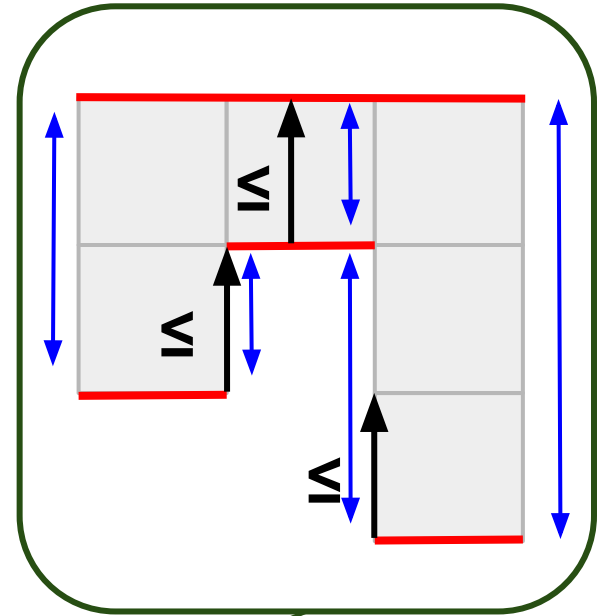
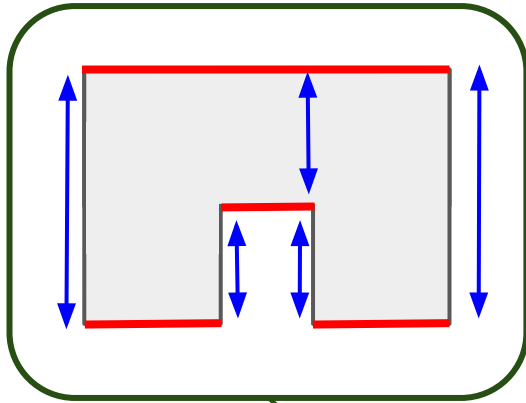
4.2 Our method: constrained optimisation

Defining our energy



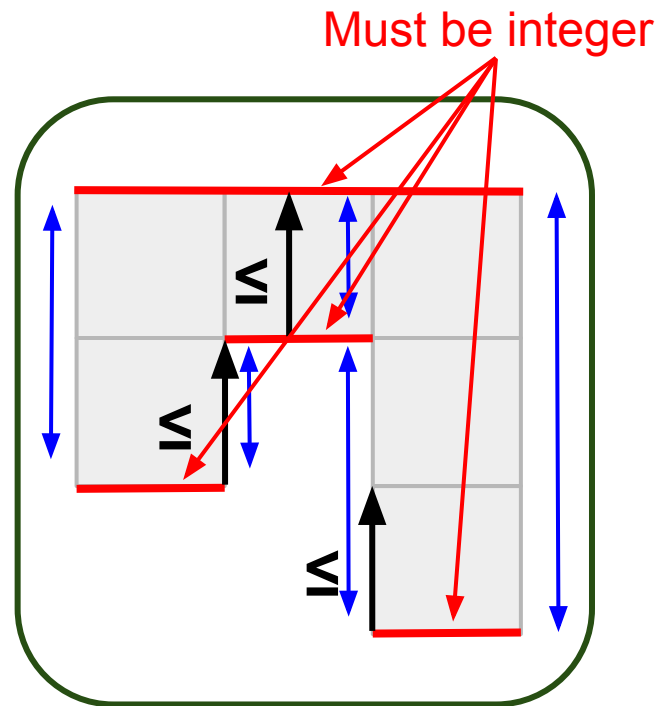
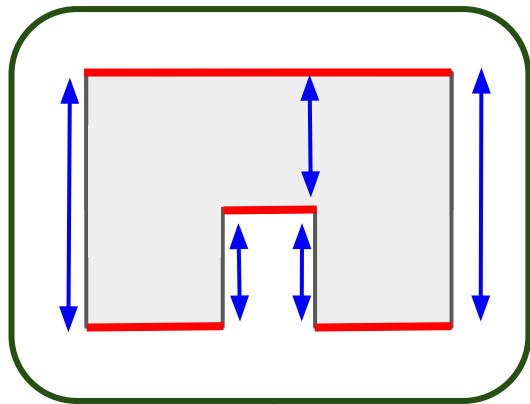
4.2 Our method: constrained optimisation

Defining our energy



We want the closest corresponding distance

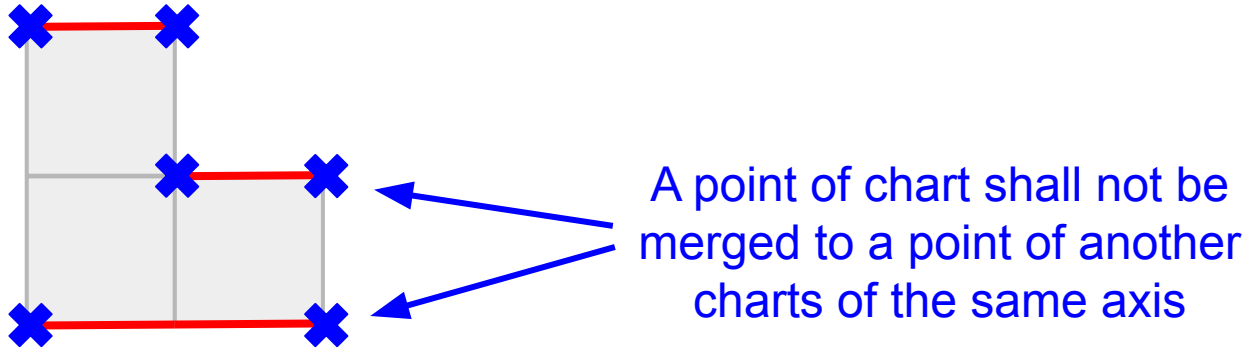
Defining our energy



We want the closest corresponding distance

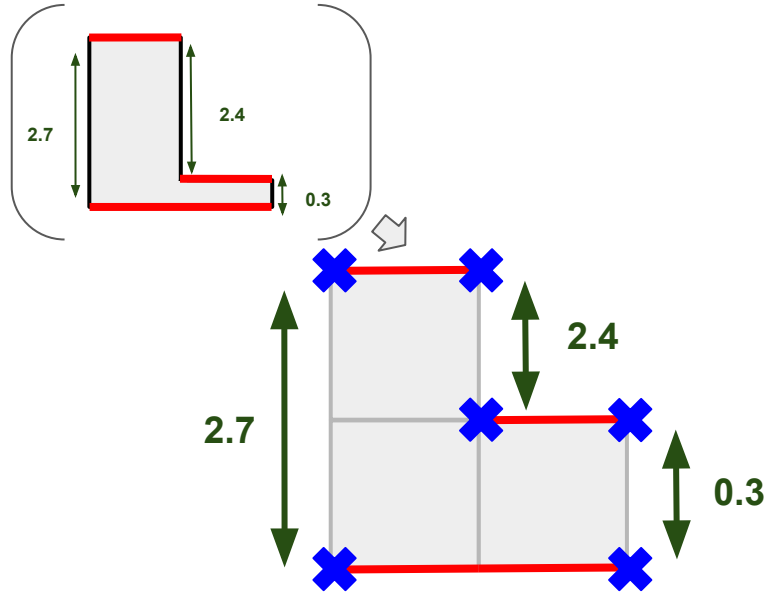
4.2 Our method: constrained optimisation

Expliciting constraints



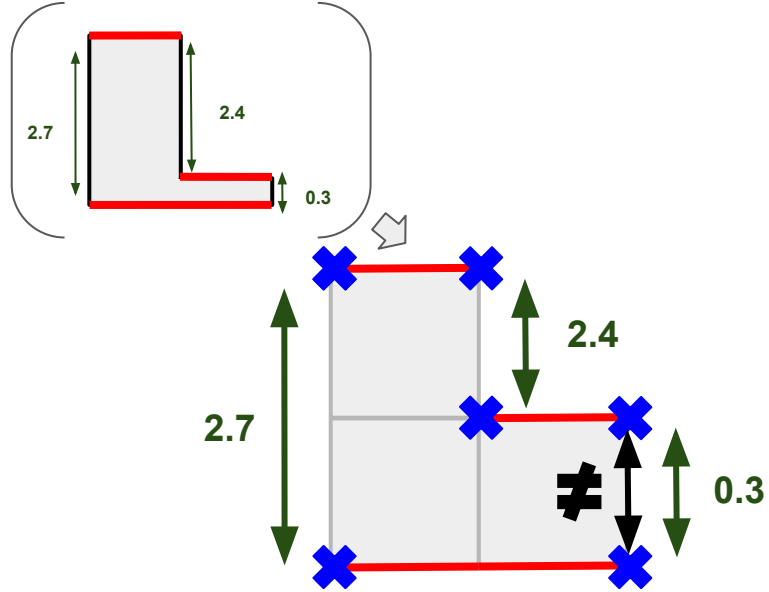
4.2 Our method: constrained optimisation

Expliciting constraints: example 1



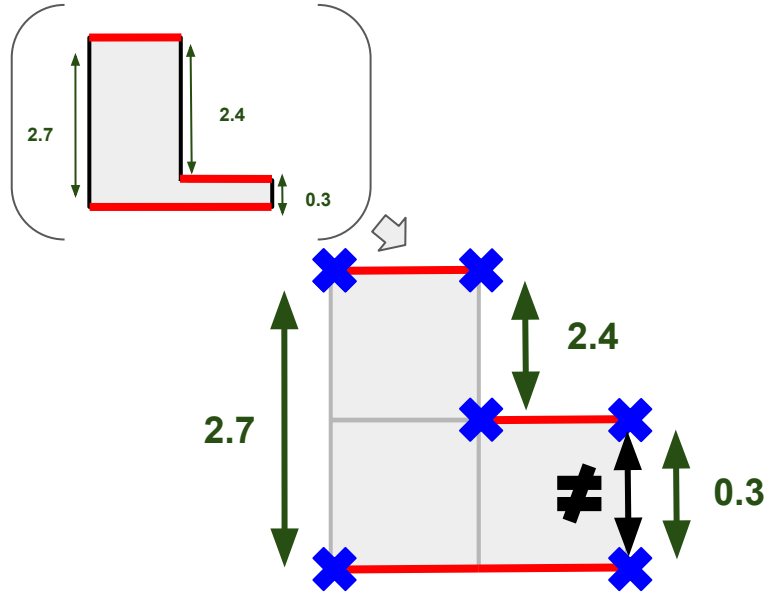
4.2 Our method: constrained optimisation

Expliciting constraints: example 1



4.2 Our method: constrained optimisation

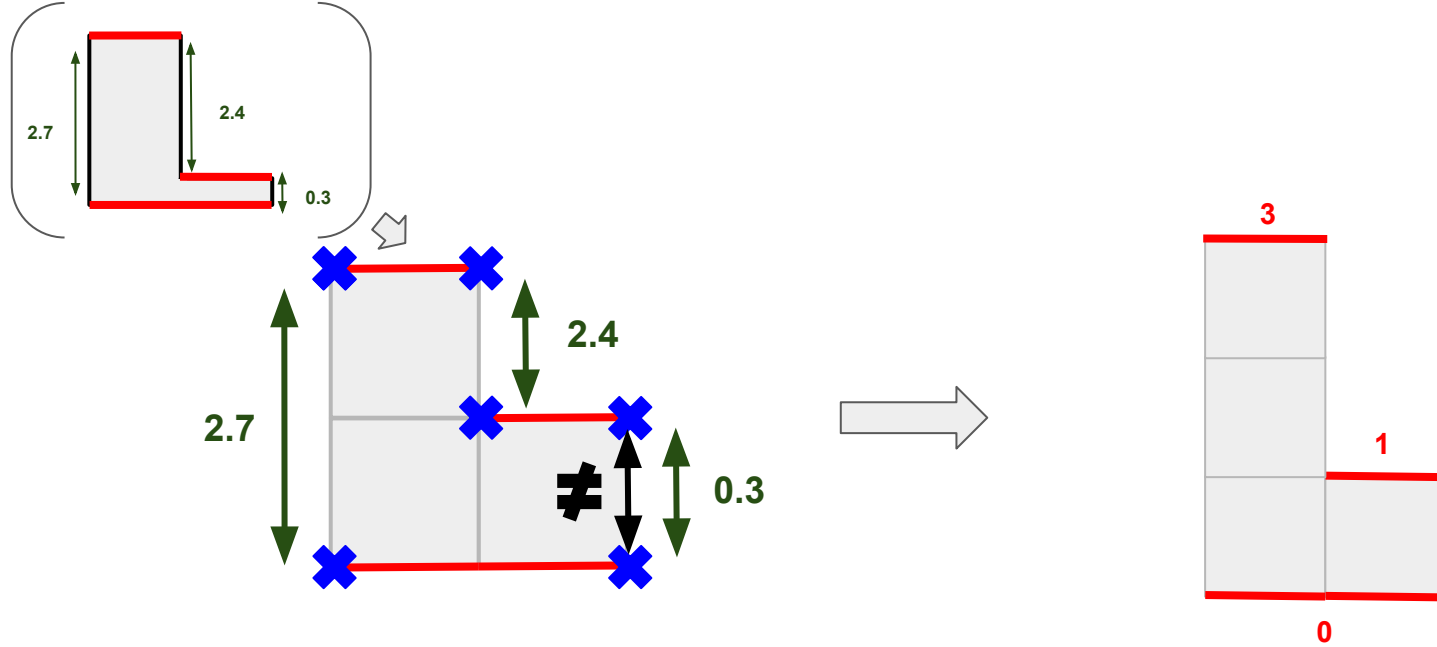
Expliciting constraints: example 1



([Chen et al, 2019])

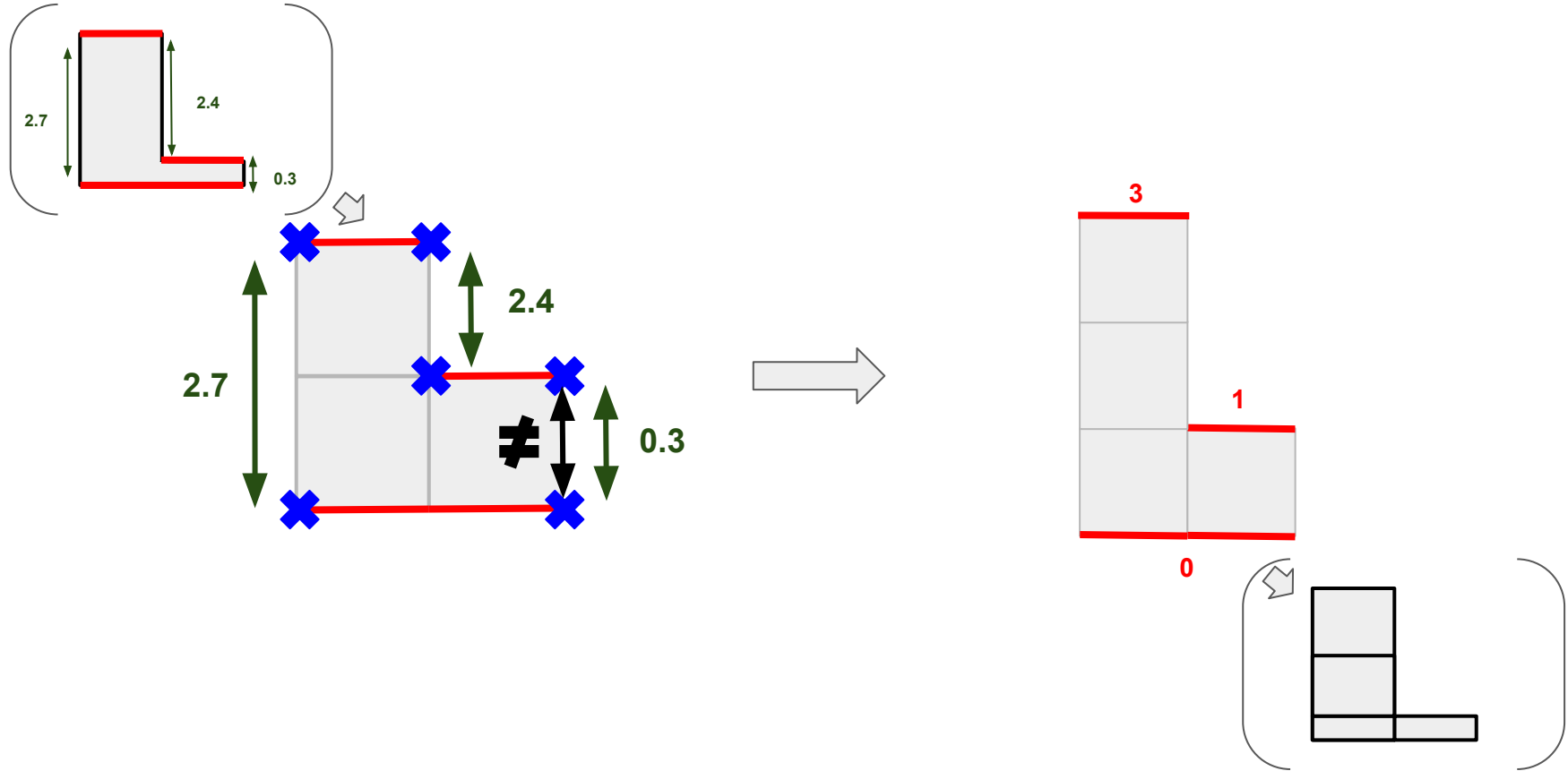
4.2 Our method: constrained optimisation

Expliciting constraints: example 1



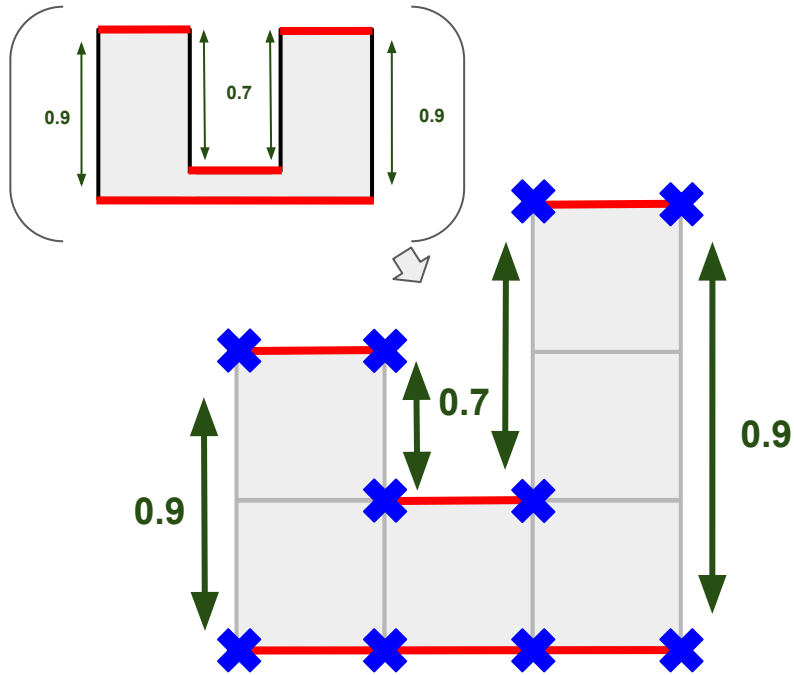
4.2 Our method: constrained optimisation

Expliciting constraints: example 1



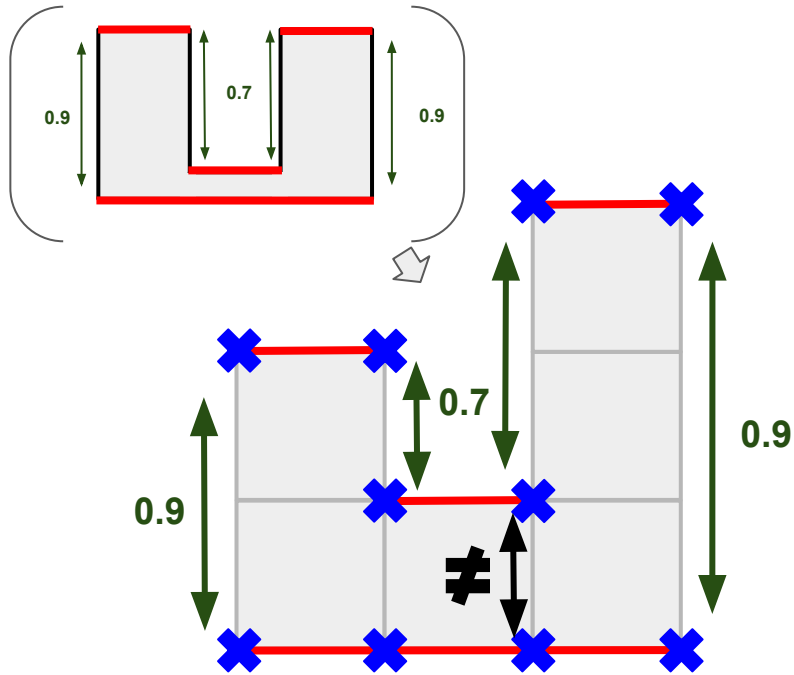
4.2 Our method: constrained optimisation

Expliciting constraints: example 2



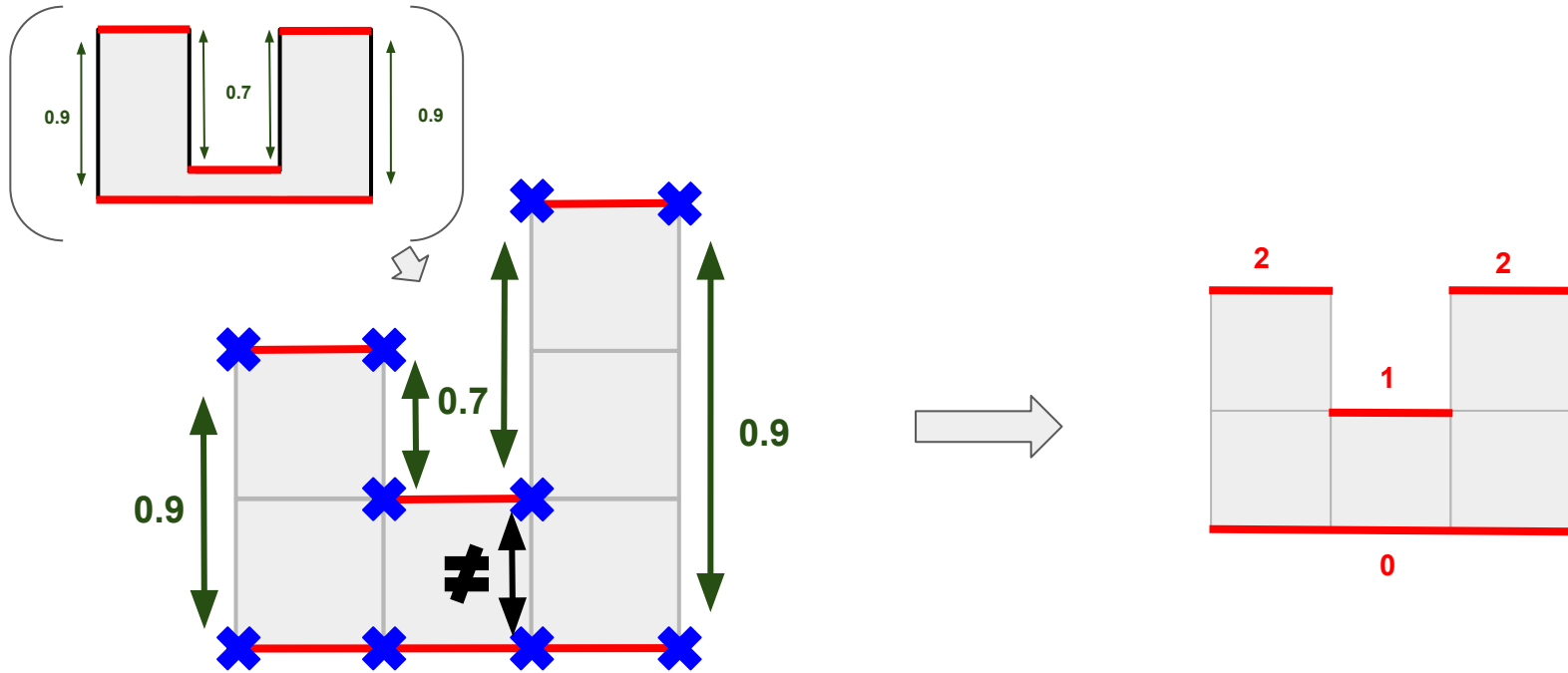
4.2 Our method: constrained optimisation

Expliciting constraints: example 2



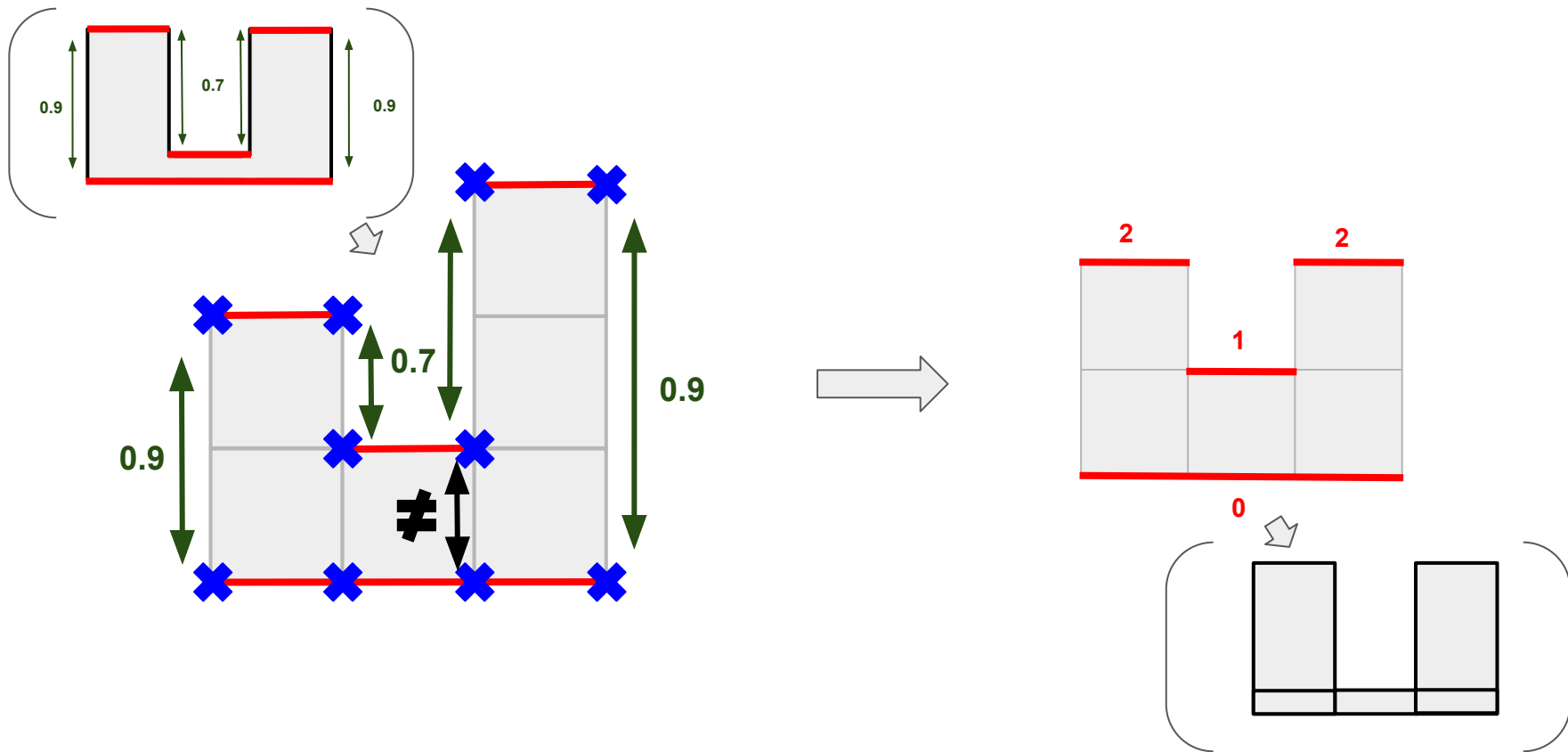
4.2 Our method: constrained optimisation

Expliciting constraints: example 2



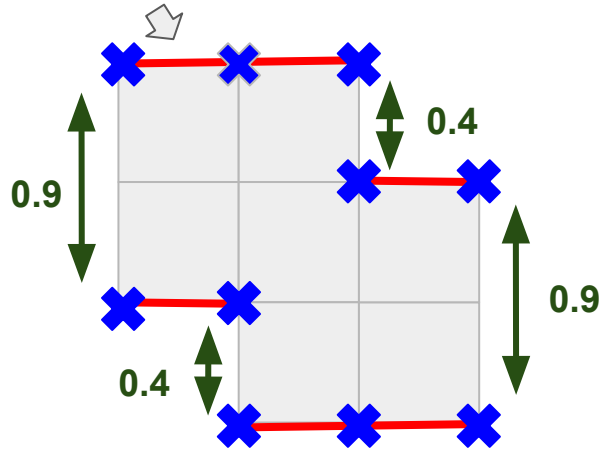
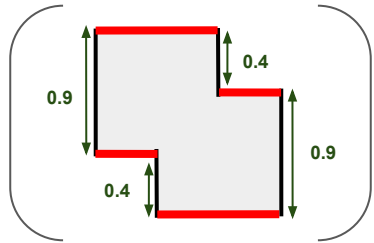
4.2 Our method: constrained optimisation

Expliciting constraints: example 2



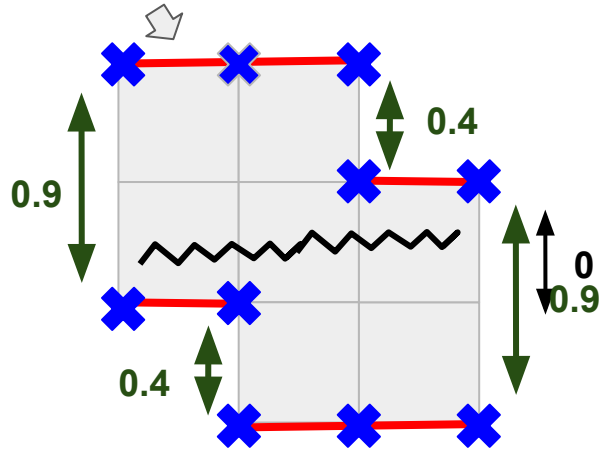
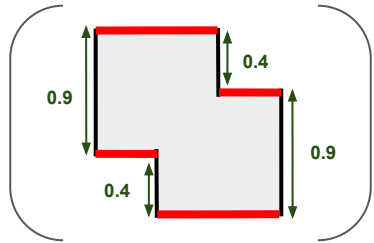
4.2 Our method: constrained optimisation

Expliciting constraints: example 3



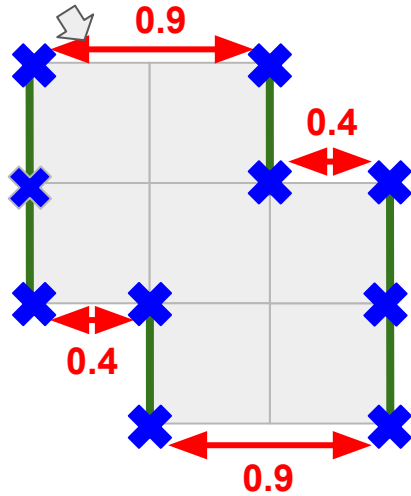
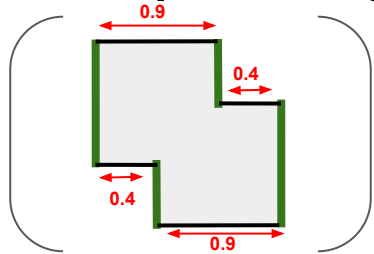
4.2 Our method: constrained optimisation

Expliciting constraints: example 3



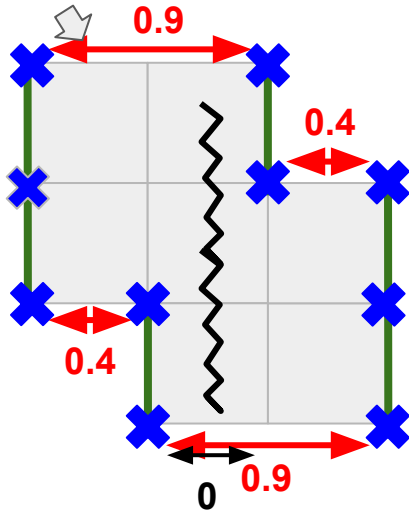
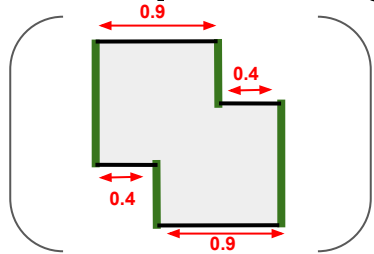
4.2 Our method: constrained optimisation

Expliciting constraints: example 3



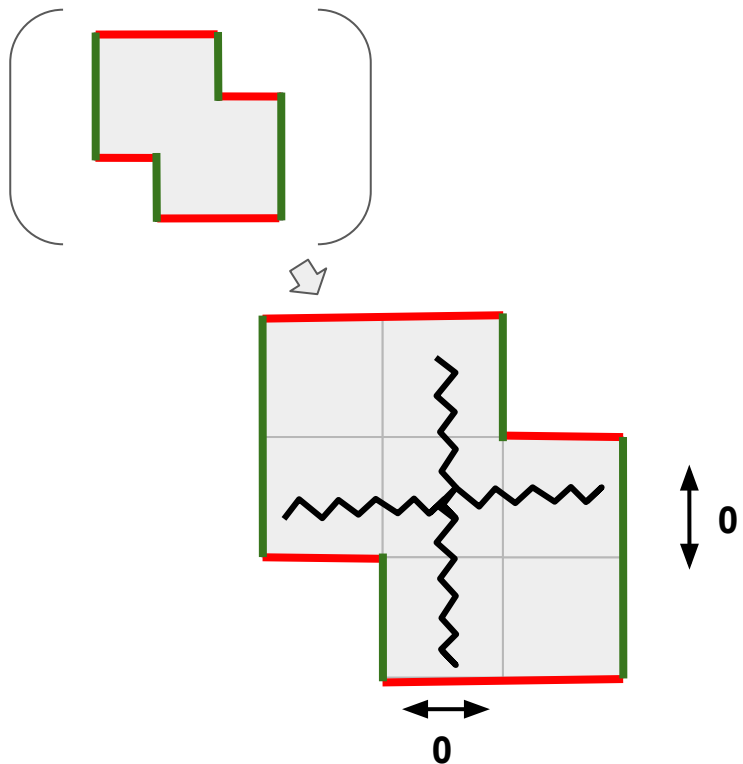
4.2 Our method: constrained optimisation

Expliciting constraints: example 3



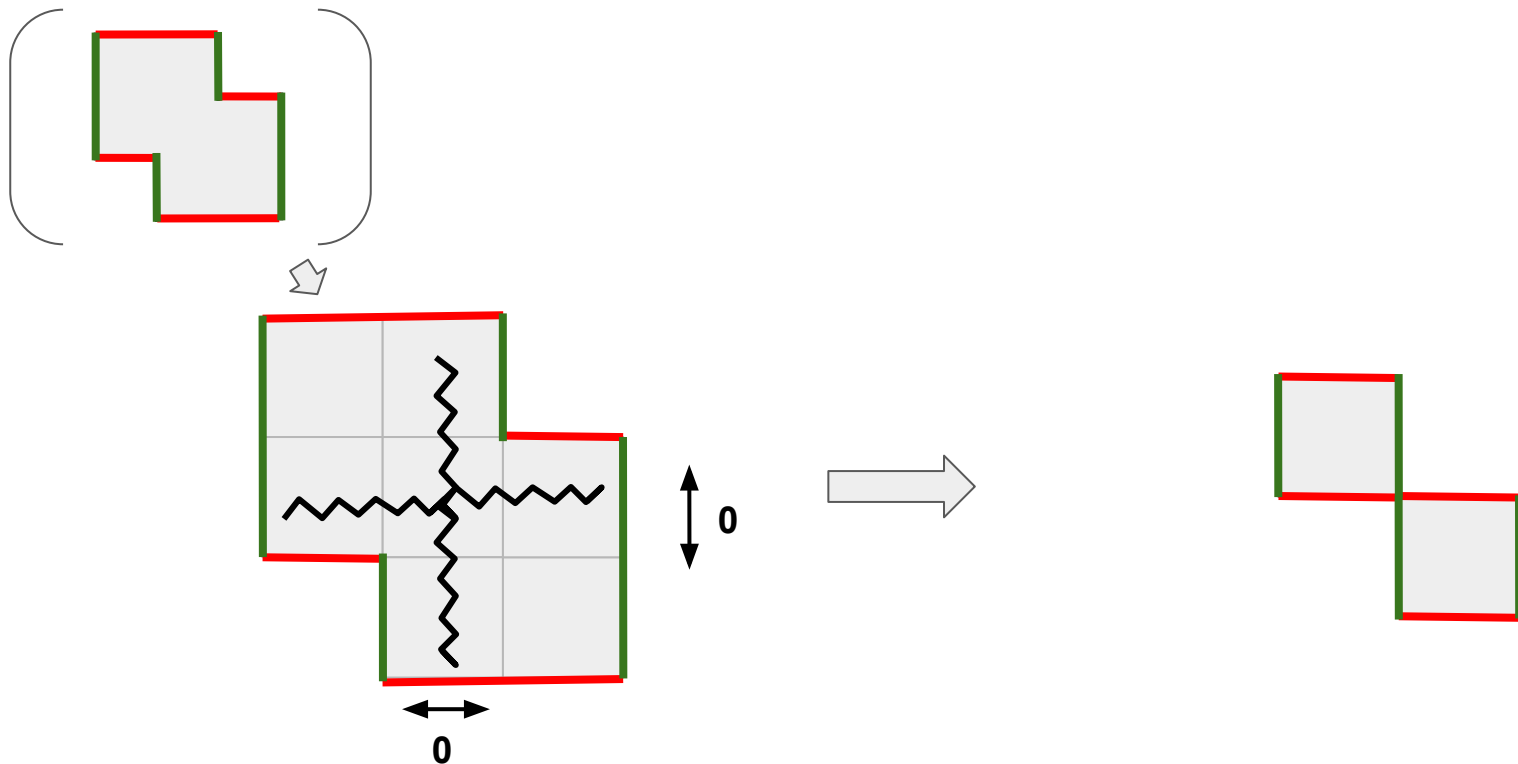
4.2 Our method: constrained optimisation

Expliciting constraints: example 3



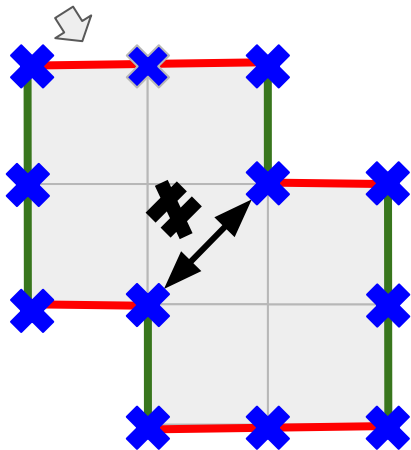
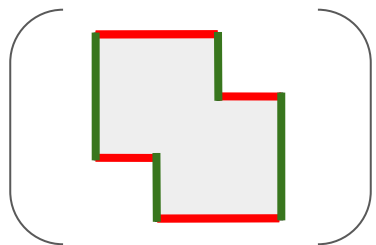
4.2 Our method: constrained optimisation

Expliciting constraints: example 3



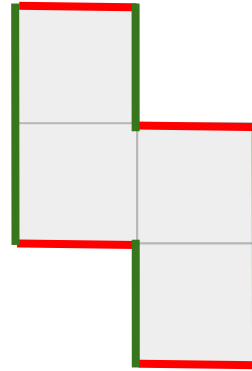
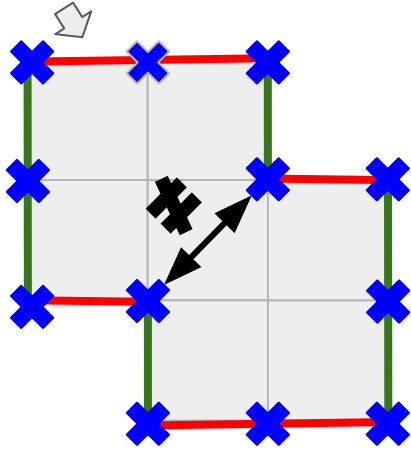
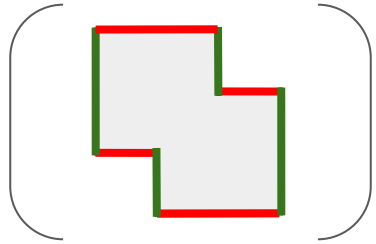
4.2 Our method: constrained optimisation

Expliciting constraints: example 3

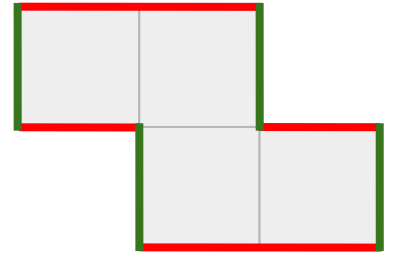


4.2 Our method: constrained optimisation

Expliciting constraints: example 3

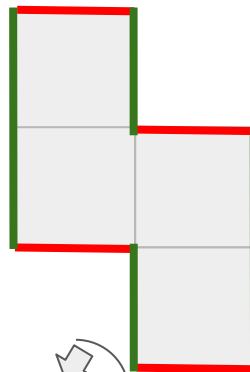
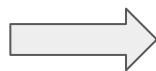
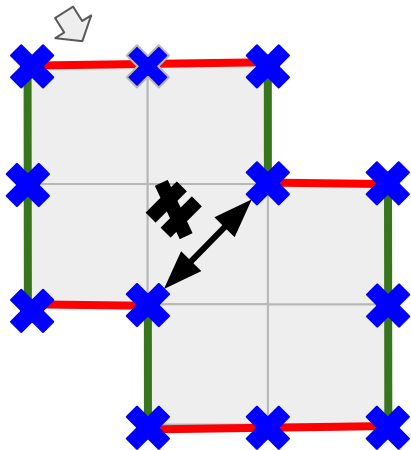
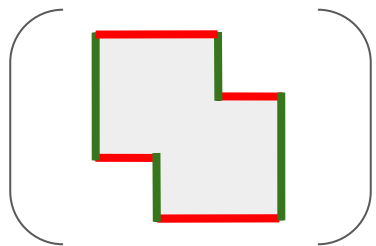


or

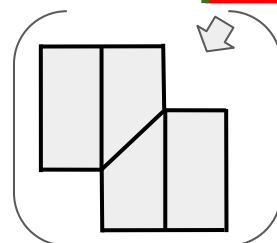
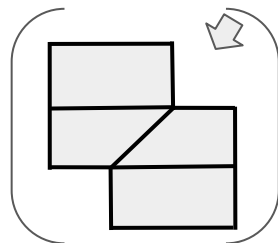
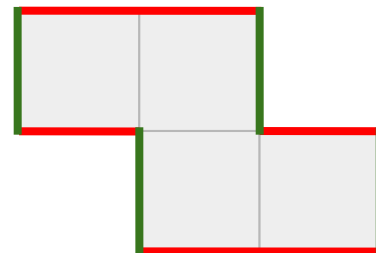


4.2 Our method: constrained optimisation

Expliciting constraints: example 3

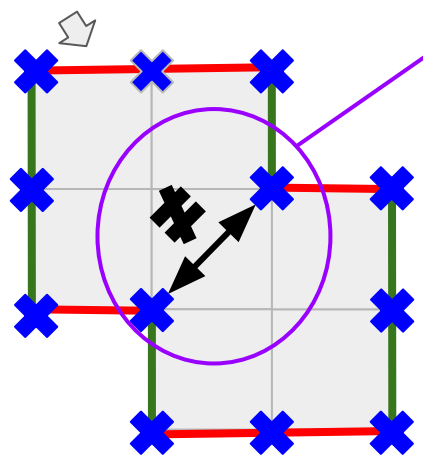
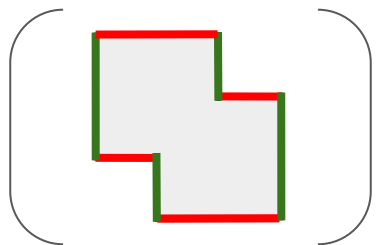


or

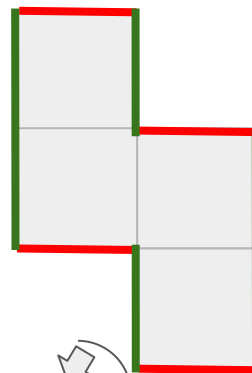
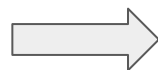


4.2 Our method: constrained optimisation

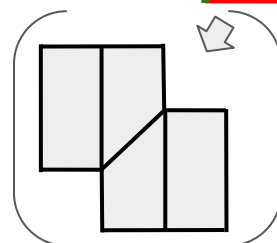
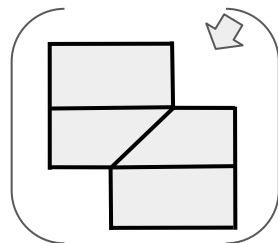
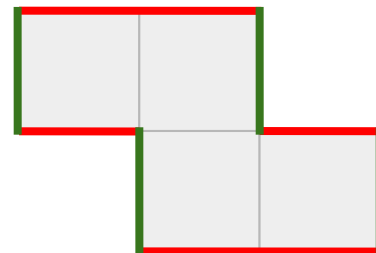
Expliciting constraints: example 3



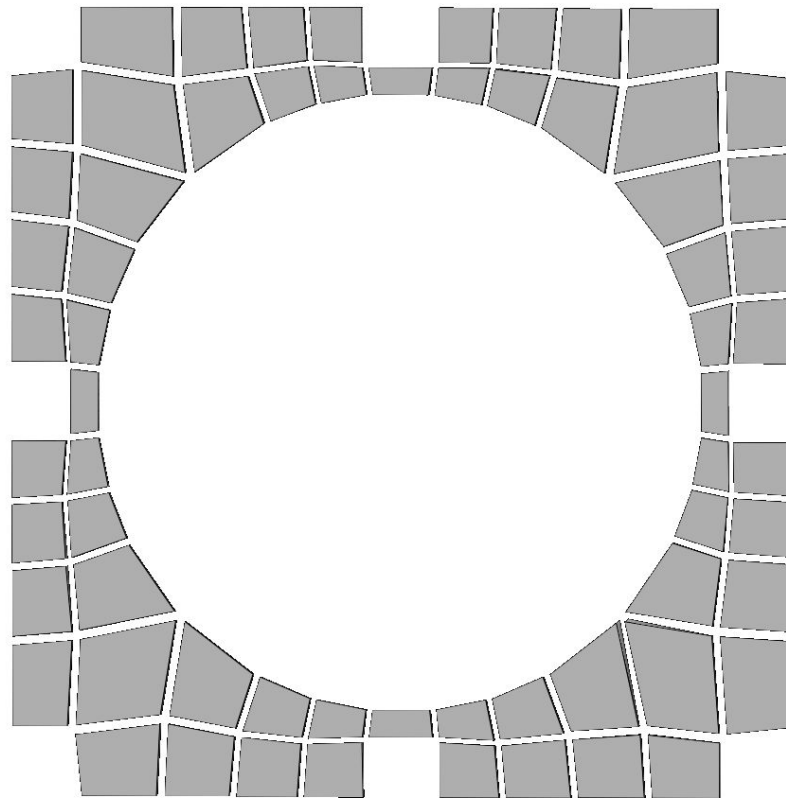
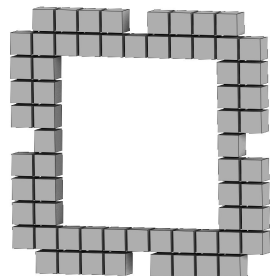
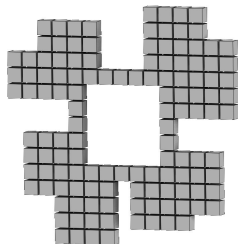
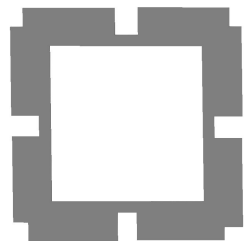
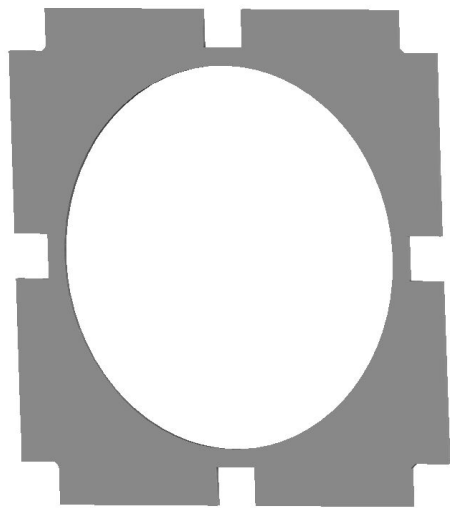
Not separable anymore



or

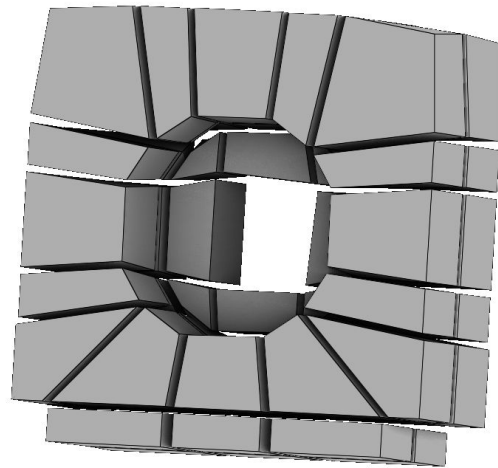
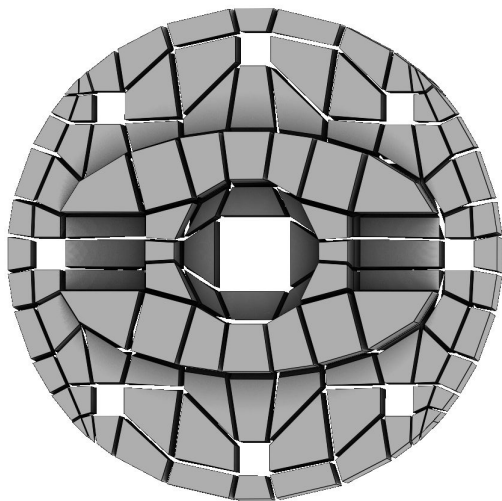
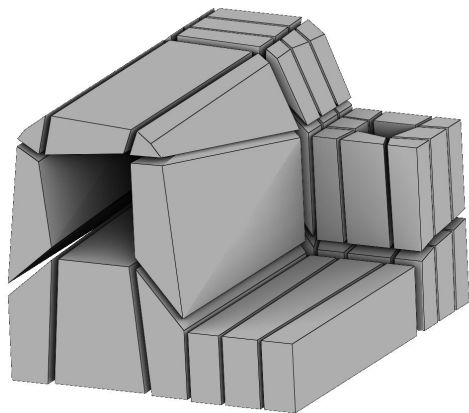
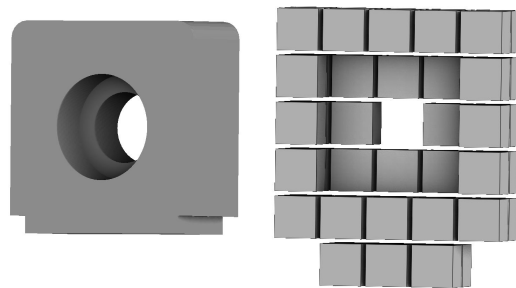
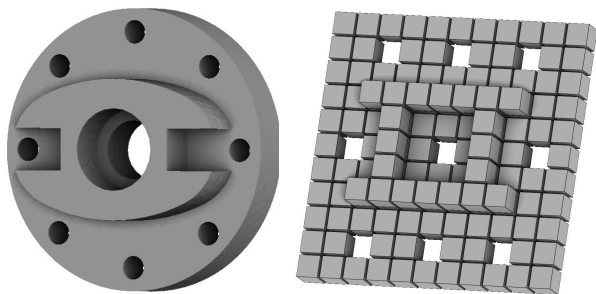
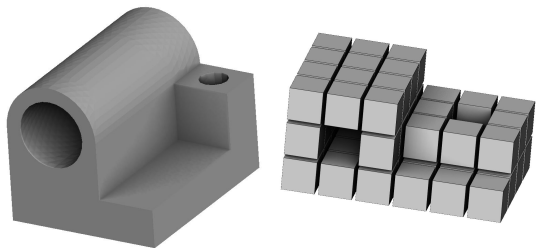


Results



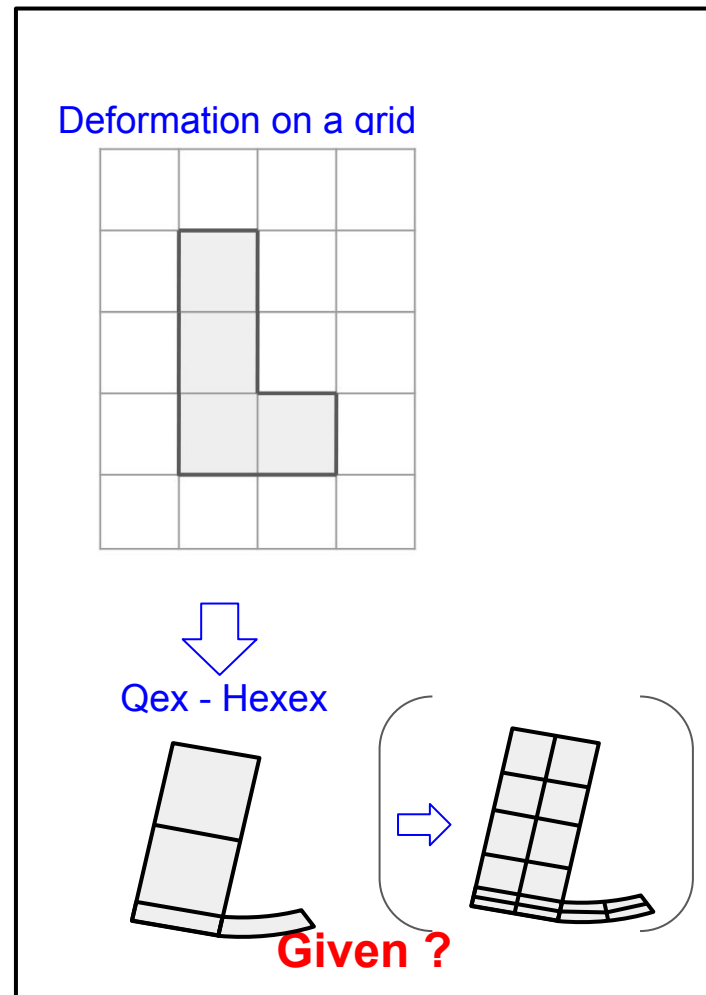
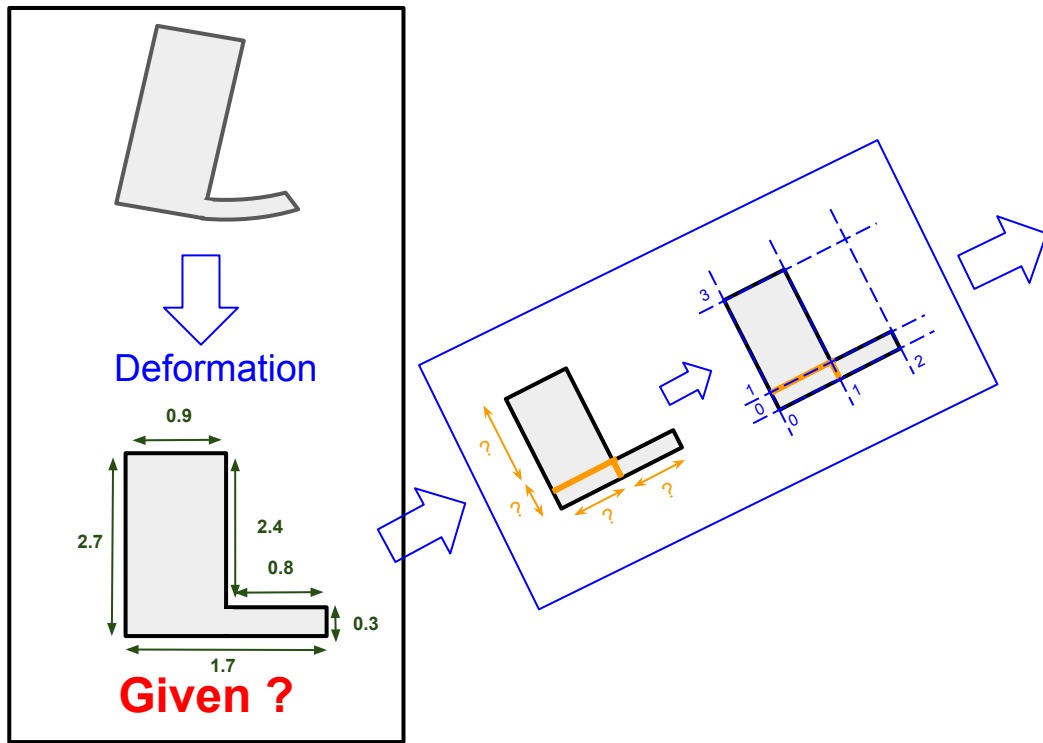
4.3 Our method: Results

Aiming for big cubes

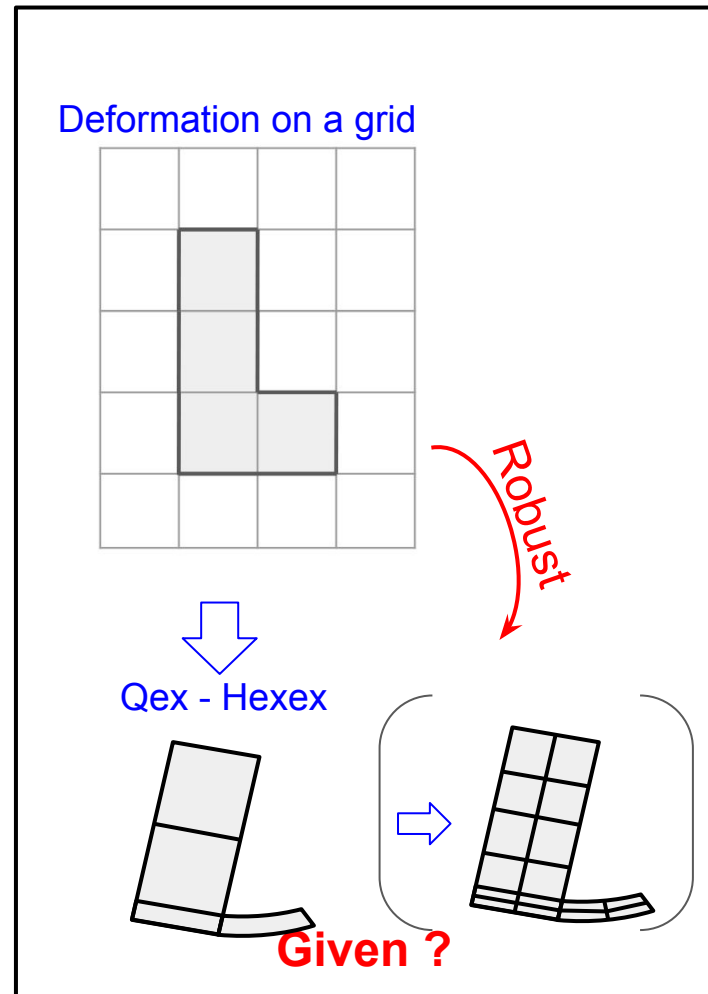
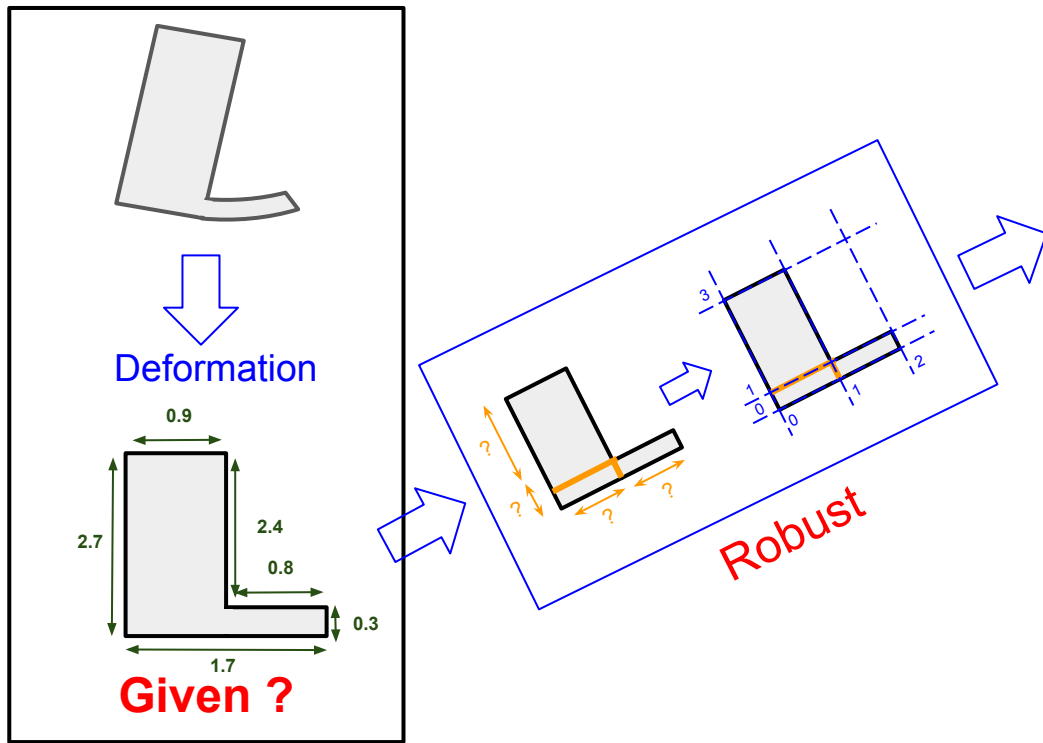


4.3 Our method: Results

Talk on robustness

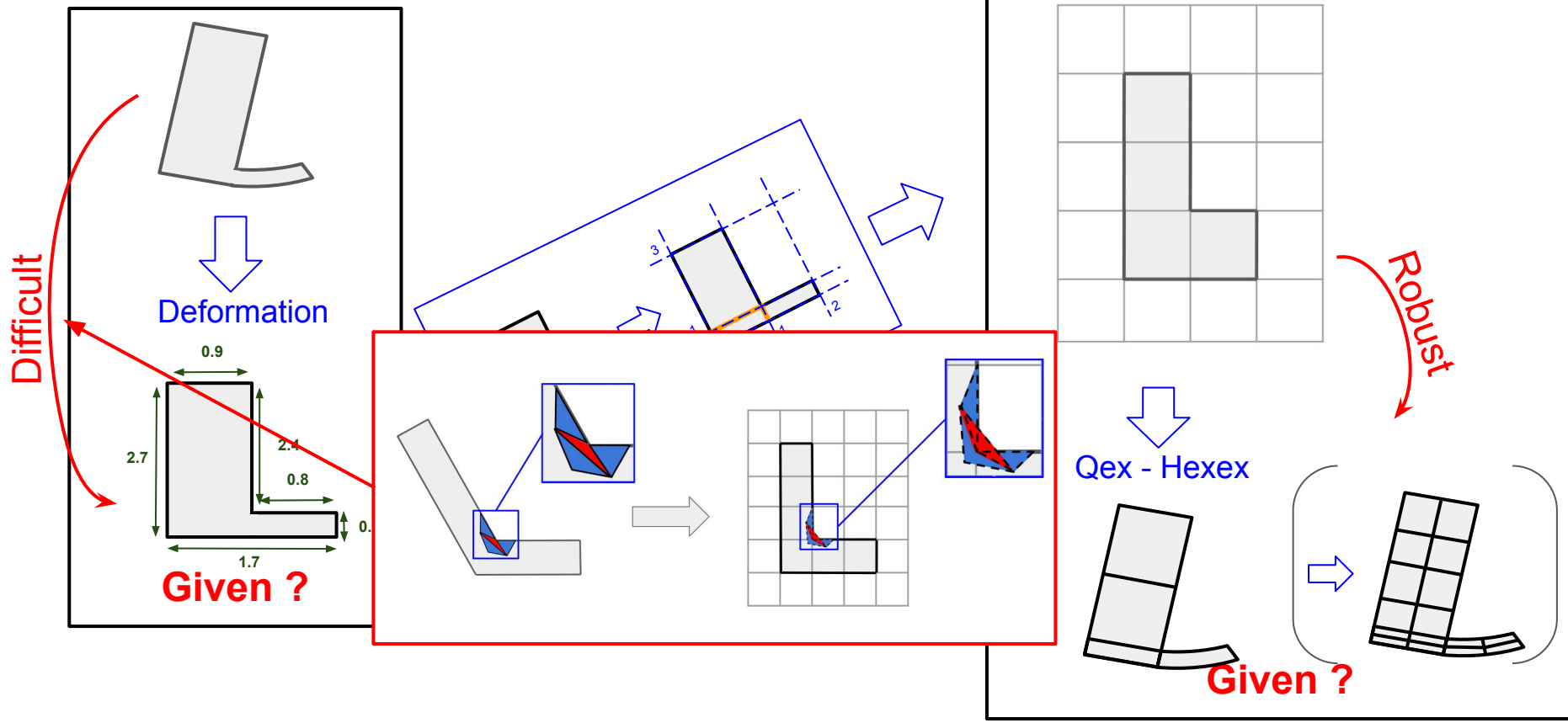


Talk on robustness

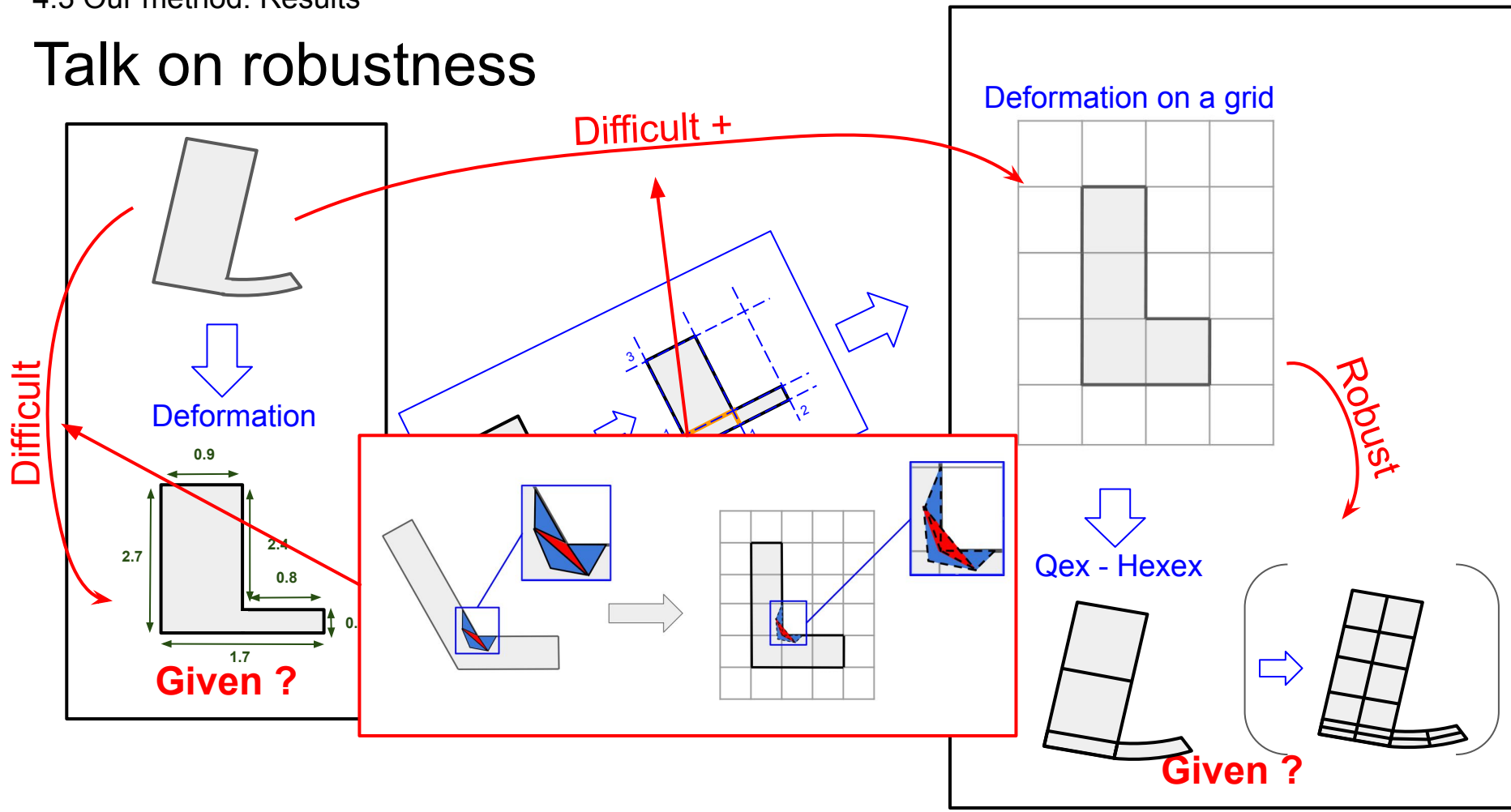


4.3 Our method: Results

Talk on robustness



Talk on robustness



Thanks for your attention

