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High quality mesh generation using cross and asterisk fields: Application on coastal domains.

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Abstract

We present a method to generate high quality triangular or quadrilateral meshes by using direction fields and a frontal point insertion strategy. The direction fields are either asterisk fields or cross fields. With asterisk fields we generate high quality triangulations, while with cross fields we generate right-angled triangulations that are optimal for transformation to quadrilateral meshes. The input of our algorithm is an initial triangular mesh and a direction field calculated on it. We present an algorithm that enables to efficiently generate the points using solely information from the base mesh. Regarding the quadrangulation process, we develop a quality criterion for right-angled triangles and an optimization process based on it. The algorithm is demonstrated on the sphere and examples of high quality triangular and quadrilateral meshes of coastal domains are presented.

Keywords: Surface meshing, Quadrilateral meshing, Direction fields, Geophysical flows, Parallel meshing

1. Introduction

This work is motivated by the increasing use of unstructured meshes in ocean modelling. Unstructured meshes offer certain advantages in the context of geophysical simulations, such as conforming to the complex geometries of coastal domains and supporting variable mesh sizes [1,2]. Moreover, for geophysical flow simulations quadrilateral meshes are desirable, since they contain twice less elements and can be aligned with the actual flow characteristics.

Here we introduce a general method to generate high quality quadrilateral and triangular meshes of surfaces and we apply it on spherical geometries. Nevertheless, the algorithm is designed and implemented as a tool that can handle triangulations of any orientable surface $\mathcal{S}$ embedded in $\mathbb{R}^3$. The input of our algorithm is an initial mesh $\mathcal{T}_0$, in particular a mesh on the sphere generated with the method proposed in [3]. The output is either a high quality triangular mesh or a right-angled triangular mesh that is transformed into a quadrilateral mesh. In [3] the point insertion strategy was based on a basic edge saturation Delaunay refinement procedure. The main contribution of this work is a new strategy to generate the vertices of the final mesh in an optimal way. By optimal we mean that

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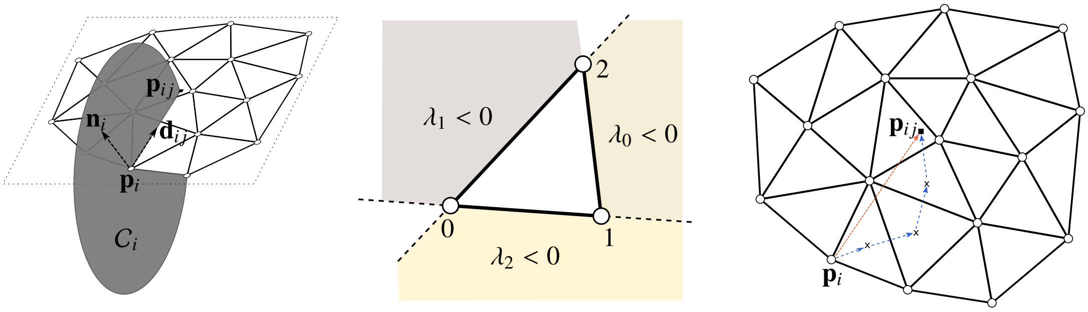

Figure 1: Computation of point $\mathbf{d}_{ij}$ (left), barycentric coordinates (center) and computation of point $\mathbf{p}_{ij}$ by walking in the triangulation(right)

the generated Delaunay triangles can be recombined in high quality quadrilaterals or that they are as equilateral as possible. Results for triangular and quadrilateral meshes on the world ocean are presented.

The method presented has been released as a self consistent open source code that can be used as a stand-alone program or that can be plugged in other software's such as Gmsh [4] or QGIS [5].

2. Frontal point insertion

To begin, we consider a base mesh $\mathcal{T}_0$ of an orientable manifold surface. The objective is to spawn points inside the domain in preferred directions. We essentially want to obtain a set of points $P = \{\mathbf{p}_1, \ldots, \mathbf{p}_n\}$ that will either (i) lead to a triangulation $\mathcal{T}_q$ that is well suited for combining triangles into quadrilaterals or (ii) lead to a triangulation $\mathcal{T}_t$ that contains triangles that are close to equilateral.

Direction fields

A *cross field* $\mathbf{c}$ is a field defined on a surface $\mathcal{S}$ with values in the quotient space S^1/Q, where S^1 is the circle group and Q is the group of quadrilateral symmetry. It associates to each point of a surface $\mathcal{S}$ to be meshed a cross made of four unit vectors orthogonal to one another in the tangent plane of the surface. In the context of quadrilateral mesh generation, a cross field represents at each point of the domain the preferred orientations of a quadrilateral mesh. It is possible to build direction fields for building $\mathcal{T}_t$ by defining asterisk fields. An *asterisk field* $\mathbf{a}$ is also a field defined on a surface $\mathcal{S}$ but with values in the quotient space S^1/H, where H is the symmetry group of a regular hexagon. Pictorially, it associates to each point of the surface $\mathcal{S}$ an asterisk made of six unit vectors separated by 60 degrees. Such a field will be used to align triangular meshes to pre-defined directions in order to build triangles that are close to equilateral. Direction fields should be as smooth as possible and should be aligned with the boundary of the domain. More details on direction fields and their computation can be found in [6].

Generating points across direction fields

Assume a direction field $\mathbf{f}$, defined everywhere on $\mathcal{S}$, that is either a cross field ($N_d = 4$ directions) or an asterisk field ($N_d = 6$ directions). A priority queue is initially filled with all the boundary points of the base mesh $\mathcal{T}_0$. The point $\mathbf{p}_i$ at the top of the queue then tries to insert N_d points $\mathbf{p}_{ij}$ in the $j = 1, \ldots, N_d$ directions defined by $\mathbf{f}(\mathbf{p}_i)$ and at a distance $h(\mathbf{p}_i)$, where h is the mesh size field. In order to have points inserted "by layers", the priority queue that is chosen is a first-in, first-out queue. Our experience shows that ordering the 1D points allows smooth propagation of the fronts inside the domain. Points therefore are ordered geometrically by sorting them along a space filling curve (Hilbert curve).

Intersection with triangulation

Assume a point $\mathbf{p}_i$ that lies on one of the triangles of $\mathcal{T}_0$, a direction $\mathbf{d}_{ij}$ i.e. a unit vector tangent to the surface and the mesh size $h(\mathbf{p}_i)$ at that point. The new point $\mathbf{p}_{ij}$ will be located at the intersection of the triangulated surface $\mathcal{T}_0$ and a circle C_i of center $\mathbf{p}_i$ and radius $h(\mathbf{p}_i)$. C_i lies on the plane $\mathcal{P}_i$ that is formed by the direction vector $\mathbf{d}_{ij}$ and the

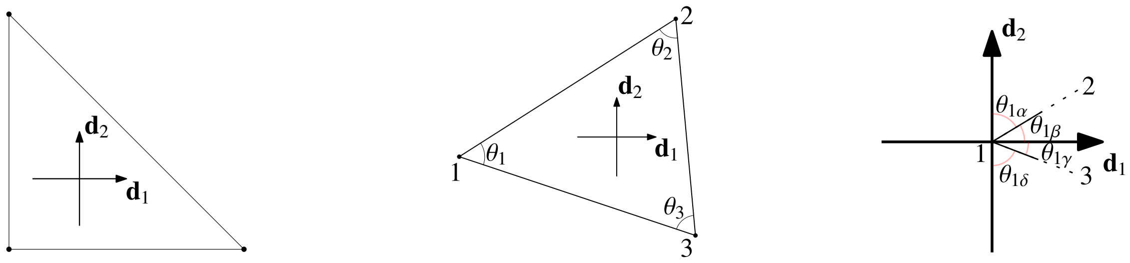

Figure 2: Optimal right-angled triangle aligned with the cross frame (left), triangle to be evaluated (middle), angles between edges of vertex 1 with the two main directions (right)

normal to the triangulation at our origin point, $\mathbf{n}_i$ (Figure 1, left). To compute $\mathbf{p}_{ij}$ our goal is to find the intersection point of circle C_i with the triangulation $\mathcal{T}_0$.

For this, we perform a walk in the triangulation [7] in the desired direction, until we obtain the intersection point (Figure 1, right). We start from the triangle of the base mesh $\mathcal{T}_{0i}$ on which $\mathbf{p}_i$ lies. First, we compute the intersection line of the plane $\mathcal{P}_i$ and the plane of the triangle $\mathcal{P}_{\mathcal{T}_{0i}}$. Then, we find the intersection points of this line with the circle C_i and choose the one that lies in direction $\mathbf{d}_{ij}$. Finally, the barycentric coordinates $\lambda_0, \lambda_1, \lambda_2$ of this point with respect to the current triangle are calculated (Figure 1, center).

In the case where the current triangle is not intersected, we move forward to a neighbour triangle. Since we have already computed the barycentric coordinates with respect to the current triangle $\mathcal{T}_{0i}$, we have an indication on which neighbour triangle to search. This procedure continues until a valid intersection point is retrieved.

Filtering

After each point $\mathbf{p}_{ij}$ is generated, a filtering procedure follows. During the point insertion process, we store on which triangle of the base mesh $\mathcal{T}_0$ each generated point lies. Therefore we are able to directly obtain the set of points $P_f = \{\mathbf{p}_k, k = 1, \ldots, n_f\}$ in the vicinity of $\mathbf{p}_{ij}$. Since our objective is to create right-angled triangles, i.e. equilateral triangles in the $\mathcal{L}_\infty$ norm, we compute the distance between the candidate point and its surrounding ones as $||\mathbf{p}_{ij} - \mathbf{p}_k||_\infty = \max\{|x_{ij} - x_k|, |y_{ij} - y_k|, |z_{ij} - z_k|\}$ The point is accepted for insertion if condition $||\mathbf{p}_{ij} - \mathbf{p}_k||_\infty > \alpha \cdot h(\mathbf{p}_{ij})$ holds for all $\mathbf{p}_k \in P_f$, where α is chosen to be 0.75.

3. Optimization of right-angled triangles for combination into quads

Our concern now is to further improve the quality of the right-angled triangles generated from cross-fields before applying a recombination procedure. We consider that the optimal triangles would be the right-angled ones in respect to the local cross frame (Fig. 2, left), since they can be combined in quadrilaterals of optimal quality. To this end, we are investigating the development of a quality criterion for right-angled triangles, based on how 'close' the triangle is to the optimal one. For each vertex of the triangle ($i = 1, 2, 3$) we calculate three corresponding local qualities, represented from the following dimensionless quantities:

(i) a quantity to evaluate how close each angle of the triangle is to 90°: $q_a^i = 1 - \dfrac{|\frac{\pi}{2} - \theta_i|}{\frac{\pi}{2}}$

(ii) a quantity to evaluate the nearest edge of each vertex to one direction of our the local cross field (Fig. 2, right): $q_b^i = \max\Big\{|\cos(2\theta_{ij})| \ : \ j = \alpha, \beta, \gamma, \delta\Big\}$

(iii) the ratio of the two edges of each vertex: $q_c^i = 1 - \dfrac{|e_{i1} - e_{i2}|}{\max(e_{i1}, e_{i2})}$

The final right-angled quality of the triangle is the maximum amongst the product of the three local qualities of its vertices: $q_t = \max\Big\{(q_a \cdot q_b \cdot q_c)^i \ : \ i = 1, 2, 3\Big\}$.

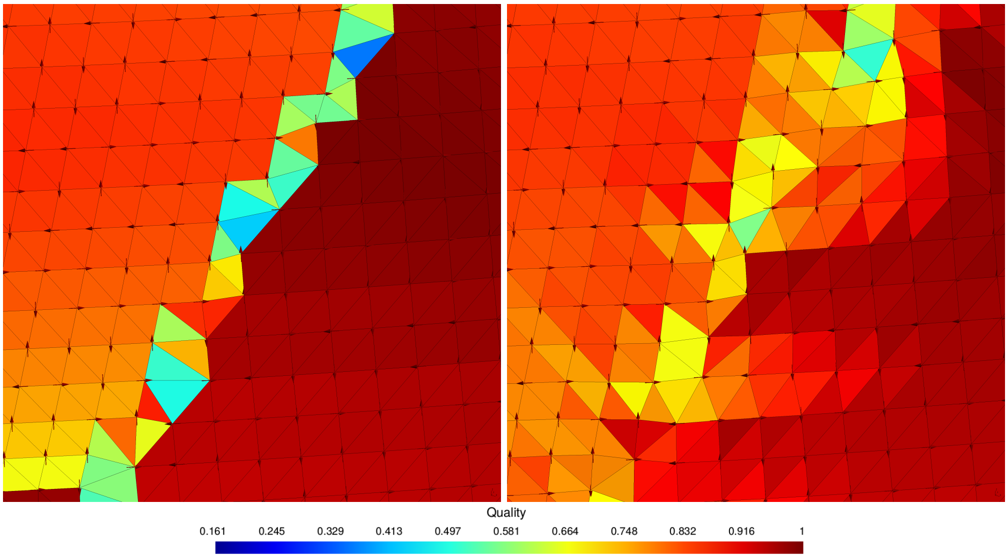

Figure 3: Quality distribution on an interface where fronts of insertion collide: before (left) and after (right) optimization.

Bad quality triangles may occur in certain regions where there is transition to different mesh sizes or where fronts of insertion collide (Fig. 3). Herein we follow a local optimization procedure to improve these regions. For each interior point of the domain we take its corresponding cavity of surrounding triangles. The minimum quality amongst the cavity's triangles is set to be the objective function to be maximized. A local maximum is then searched in the line connecting the original position of the point and the centroid of the cavity in the infinity norm, and the point is relocated accordingly.

4. Results and conclusions

In Figure 4 we present a comparison between the base and the output mesh on Baltic sea, choosing a high resolution mesh size field: from $h_{min} = 150m$ on the coast to $h = 3km$ away from it. The output triangulation exhibits improved quality and alignment of elements. Finally, we utilize our algorithm to generate a quadrilateral mesh of the whole world ocean (Figure 5), with a resolution from h_{min} = 3km to h_{max} = 60km.Generation of the final 2,267,738 points takes around 30 seconds. The final merging to quadrilaterals is done with the blossom-quad method [8] along with quadrilateral smoothing procedures incorporated in Gmsh [4].

The method presented is an efficient tool that can be used either to re-mesh existing triangulations to finer resolution and improved quality or to generate, along with the optimization procedure, right-angled triangulations suitable for recombination to high quality quadrilateral meshes. A multi-threaded strategy for the algorithm will be presented in the future. The following step of our work is the generation of boundary layer meshes by inserting points in an anisotropic fashion.

Acknowledgements

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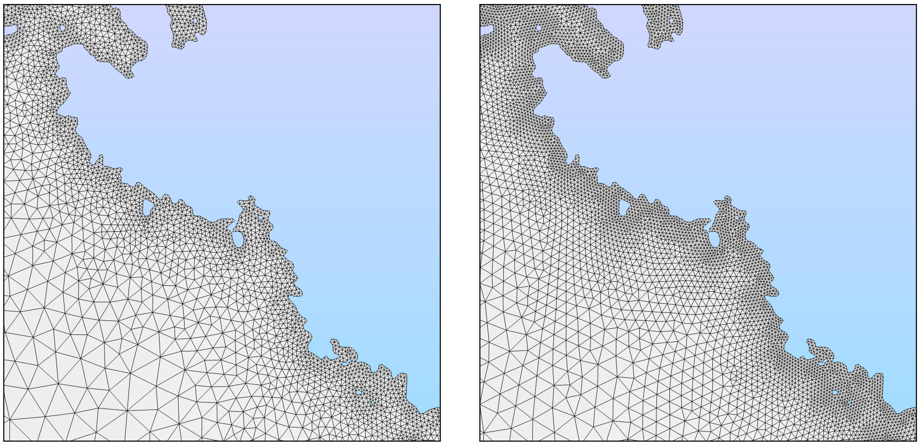

Figure 4: Images of base (left) and output (right) triangulation on the Baltic sea. Average radius-ratio quality is $\overline{\gamma} = 0.947$ ($\gamma_{min} = 0.104$) for the initial mesh. The output mesh improves it to $\overline{\gamma} = 0.981(\gamma_{min} = 0.3231)$.

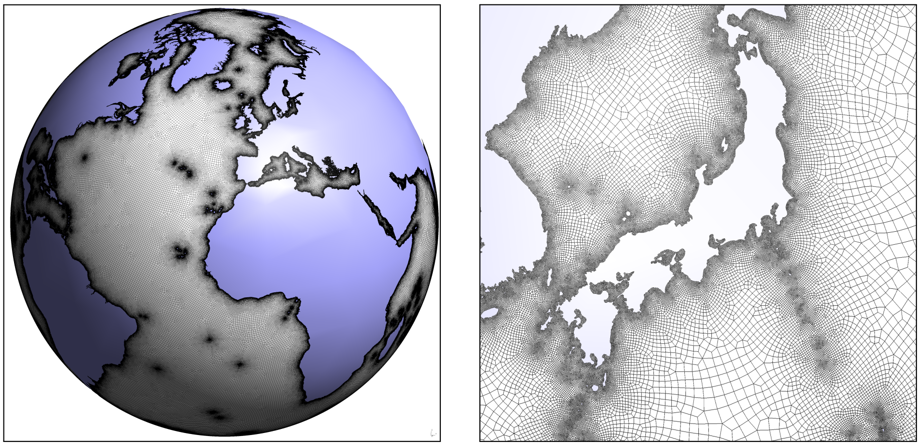

Figure 5: Quadrilateral mesh of world ocean (left) with a zoom on the sea of Japan (right). The average isotropy measure quality [9] of the output quadrilateral elements is 0.944.

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